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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
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SEPTEMBER 1917

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THE
ASTROPHYSICAL JOURNAL
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AND ASTRONOMICAL PHYSICS

VOLUME XLVI SEPTEMBER 1917 NUMBER 2

THE MATHEMATICAL STRUCTURE OF BAND SERIES[1]

By RAYMOND T. BIRGE

INTRODUCTION

It is generally agreed[2] at the present time that the mathematical relationships found in band series are conditioned primarily by molecular structure, while those of line series are due mainly to atomic structure. An exact knowledge of the structure of spectral series is therefore considered of the utmost importance in the study of the ultimate constitution of matter.

This paper is concerned exclusively with the relationship between the individual lines of a single band series, i.e., with that relationship for which the first Deslandres' law is a first approximation. An admirable review of the entire subject of regularities in band spectra is given by Konen,[3] and the "cause for action" in this field is there stated very clearly (p. 272): "Keine der bisher aufgestellten Formeln stellt die längeren, bisher untersuchten Serien genau dar." Ritz[4] has called attention to the apparently anomalous structure of very long series and to the great difficulty

[1] Presented to the American Physical Society, April, 1917.

[2] For full discussion and references see Konen, *Das Leuchten der Gase und Dampfe*, pp. 339–366; also, in particular, Königsberger, *Astrophysical Journal*, **35**, 139, 1912, and Whittaker, *Proc. Roy. Soc.* (A), **85**, 262, 1911.

[3] *Ibid.*, pp. 199–278. [4] Ritz (Weiss), *Astrophysical Journal*, **35**, 75, 1912.

of finding for them any formula, empirical or otherwise. It might be added also that those formulae which are least in error are of a purely empirical type, and not at all adapted for formulating any theory of molecular structure. The situation in the case of line series is very different, for, while the exact *functional form* of the equation is still unknown, the very fact that the frequency can be expressed as a difference of two functions ("combination principle"), taken in connection with considerations as to energy leads directly to a possible theory of atomic structure.[1] In the present paper there is presented a formula for band series which not only holds with the greatest possible accuracy for the longest known series, but which, it is hoped, is of a type suitable for application to a theory of molecular structure.

In 1886 Deslandres[2] stated his now famous law for band series, as follows: "Les intervalles d'une raie à la suivante, calculés en nombres de vibrations ou inverse de longueurs d'onde sont à peu prés en progression arithmétique." If $\Delta\nu$ expresses this first frequency-difference, we then have as the algebraic form of the foregoing law

$$\Delta\nu_m = s + mr, \qquad m = 0, 1, 2, 3, \text{ etc.} \tag{1}$$

Also, if the difference of any two successive $\Delta\nu$'s is taken, we have

$$\Delta\nu_{m+1} - \Delta\nu_m = r = \text{constant.} \tag{2}$$

The fact that the second frequency-difference is constant is the most general and concise statement of Deslandres' law, and any series for which the second difference *is* constant therefore obeys the law. This fact has not always been recognized,[3] and some confusion over a very simple matter has thereby resulted.

As shown by (1) and (2), the first frequency-difference is given by the general equation of the first degree in m, and the second difference by that of zero degree. Conversely, the frequencies themselves are given by the general equation of the second degree:

$$\nu_m = \nu_0 + am + bm^2 \tag{3}$$

or

$$\nu_m = A + B(m+c)^2. \tag{3'}$$

[1] See, for instance, Millikan, *Science*, **45**, 321–330, 1917.

[2] *Comptes rendus*, **103**, 375, 1886.

[3] I.e., Lester, *Astrophysical Journal*, **20**, 81, 1904.

(3′) is the most convenient form for reference, since the constant c (the "phase") plays an important rôle in Thiele's theory of band series.

The exact relationship between the constants in (3) or (3′) and those in (1) depends on the assumptions made regarding the values of m to be used to express each ν and each $\Delta\nu$. The most convenient assumptions in this regard are the following: that the frequencies of successive lines shall be given in (3) or (3′) by the succession of integral values $m=0, 1, 2, 3$, etc., while the corresponding first frequency-differences shall be given in (1) by $m=0.5, 1.5, 2.5$, etc., i.e.,

$$\Delta\nu_m = s + mr, \qquad m = 0.5, 1.5, 2.5, \text{ etc.} \tag{1′}$$

Thus

$$\nu_4 - \nu_3 = \Delta\nu_{3.5}.$$

By these assumptions the $\Sigma\Delta\nu_m$ becomes identical with the $\int\Delta\nu_m$. For, if the frequency of the first line $[m=0]$ is called ν_0, we have for the frequency of the $(n+1)$th line $[m=n]$

$$\nu_n = \nu_0 + [\Sigma\Delta\nu_m]_{m=0.5}^{m=n-0.5} = \nu_0 + sn + r/2\, n^2. \tag{4}$$

This is identical with the integral of (1′). A comparison of (4) with (3) and (3′) gives the relations between the various constants

$$[r/2 = B = b;\ c = a/2b = s/r;\ \nu_0 = A + Bc^2].$$

Equation (1′) is that of a straight line with the intercept $m=-c$, while (4), (3), and (3′) represent a parabola with the vertex at $m=-c$, $\Delta\nu=A$ (see Fig. 1). I call the equation (1′) the "slope" form, since it is identical with the equation in terms of the slope of the parabola $d\nu/dm$ and m, the integral of which gives the original parabola.[1] In general, however, the $\Sigma\Delta\nu$ will not be identical with the $\int\Delta\nu$, for, while the integral gives the true area under the $\Delta\nu$-m curve (from $m=0$ to $m=n$), the $\Sigma\Delta\nu$ gives the

[1] In respect to the values of m to be applied to ν and $\Delta\nu$, Deslandres made the same assumptions as are expressed by (1′), but instead of having the corresponding frequencies given by $m=0, 1, 2, 3$, etc., he used $m=1, 2, 3, 4$, etc. Therefore in (4) the "n" should be replaced by $(n-1)$, in order to obtain the exact equation used by him (*ibid.*). By Deslandres' conventions the parabola, instead of having its vertex directly below the intercept of the straight line (Fig. 1) would be shifted one unit to the right, and (1′) is no longer the derivative of the frequency equation, thus complicating the computations.

area under a series of chords drawn along the Δv-m curve, joining the points corresponding to $m=0.5,\ 1.5,\ 2.5\ \ldots\ (n-0.5)$, *plus* the two rectangles (one at each end) $\frac{1}{2}\Delta v_{0.5}$ and $\frac{1}{2}\Delta v_{n-0.5}$. It is of course only because the arc and chord coincide for a straight line that we can write

$$v=v_0+\Sigma\Delta v=\int\Delta v.$$

The importance of this point will appear later.

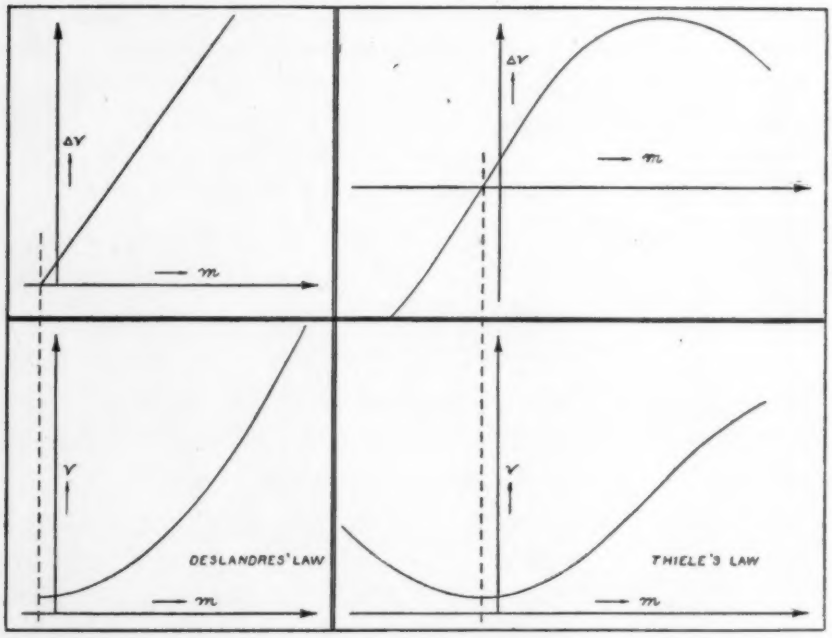


FIG. 1

Now there is nothing in Deslandres' law to specify what particular integral value of m shall be assigned to any particular line of a series. In other words, the straight line and parabola of Fig. 1 can be shifted at will parallel to the m-axis, the vertex of the parabola, however, always remaining under the intercept of the straight line. But it is customary to make this intercept (c) as small as possible, i.e., equal to 0.5 or less. Under these conditions it is found that substituting $n=0$ in (4) usually, *but not always*, gives the frequency of a definite, observable "head." The only

reason, in the author's opinion, why $n=0$ does not always give the observed head is that many series begin with zero intensity, and the observed head belongs to some other series which happens to begin with great intensity. It seems best to *define* the head of a series as that frequency obtained by putting $m=0$ in any suitable frequency-equation (shifted, as explained above), regardless of whether or not there is any observable radiation at the predicted location. This matter will be discussed in more detail in another paper.

Deslandres found that when he had thus shifted the origin so as to make the intercept in (1′) as small as possible, he might equally well *assume* this intercept to be zero. In fact, even for that small portion of a band series for which Deslandres' law is practically true, the wave-lengths must ordinarily be known with certainty to at least 0.005 A in order to get any determinate value of c. With the data at Deslandres' disposal, the appropriate values of m were usually indeterminate to one or more whole units. Deslandres therefore *actually used* his law in the *approximate* form

$$\nu_m = A + Bm^2. \qquad (3'')$$

This, however, is *not* Deslandres' law, for it assumes in (1′) the special relationship $s=0$, which is not assumed in any way in the original statement of the law.[1] Thiele found that for some series the phase $c=0$. In general it did not. In other words, the shifted $\Delta\nu$-m curve sometimes seems to head straight for the origin, but in general it does not.

As was recognized by Deslandres himself, the foregoing law (3) is only a first approximation, applying at best to a short portion, near the head, of a band series. For some series it will hold apparently for nearly 60 members, in others, for a much smaller number. The actual form of the $\Delta\nu$-m and of the ν-m curve for the longest known band series (those of the CN 3883 band) is given roughly by the portion of the curves to the right of the vertical axis in the right half of Fig. 1. The first frequency-difference, instead of increasing linearly, actually attains a maximum value, corresponding to a point of inflexion on the ν-m curve.

[1] Deslandres, *op. cit.*; see also *Comptes rendus*, **138**, 317, 1904.

Thiele[1] attempted to explain these facts, and many others, by assuming that the general law for band series is

$$\nu = f[(m+c)^2], \tag{5}$$

where c = constant, and was called by Thiele the "phase" of the series. The Thiele theory has been definitely disproved in every essential point in two articles, one by H. S. Uhler[2] and the other by Uhler and Patterson.[3] The writer could give additional evidence in this matter, but it seems superfluous.

The essential points in the Thiele theory are shown by the two right-hand curves of Fig. 1. In equation (5) the series obtained by using *negative* integral values of m (negative branch) is assumed to exist, as well as that corresponding to positive values (positive branch). Thus the ν-m curve is symmetrical with respect to the line $m = -c$, and both branches of the curve represent series. If $c = 0$ or $\frac{1}{2}$, the lines of the two series coincide. If $c = \frac{1}{4}$, the lines of one series fall approximately (*not exactly*) halfway between those of the other series. The simple fact that in many cases the two series which leave the same "head" of a band actually cross over one another is sufficient proof against the validity of Thiele's hypothesis.

The other essential point in Thiele's theory is that, while $m = 0$ gives the "head" of a series, $m = \infty$ gives the "tail." This indicates that the ν-m curve becomes asymptotic to the horizontal line ν_∞, while the $\Delta\nu$-m curve becomes asymptotic to the m axis. The tail of a band series is thus identical in structure with the head of a line series. *No tails of the structure assumed by Thiele have ever been found.* The so-called tails of the CN bands, first measured by King,[4] either have no structure at all or else have the structure of an ordinary band head, and are thus composed of a finite number of lines instead of an infinite number. Uhler and Patterson[5] and others[6] make this point very clear, and their conclusions have been verified by the author, using his own negatives. Thus there is at the present time no proven theory concerning the structure of band series.

[1] *Astrophysical Journal*, **6**, 65, 1897; **8**, 1, 1898.

[2] *Ibid*, **42**, 72, 1915.

[3] *Ibid.*, **42**, 434, 1915.

[4] *Ibid*, **14**, 323, 1901.

[5] *Ibid.*

[6] Ritz (Weiss), *ibid.*; Strutt and Fowler, *Proc. Roy. Soc.* (A), **86**, 105, 1912.

It might be well to state that, in searching for new spectral formulae, it is very convenient to work with the $\Delta\nu$-m ("slope") curve, since its equation must necessarily be of degree one lower than the final equation in frequency, and must have one less arbitrary coefficient.[1]

EXPERIMENTAL MATERIAL

When this investigation was started, it appeared, from the literature, that the A_1 series of the 3883 CN band was not only the longest band series known, but also that series which showed the most radical deviations from Deslandres' law. It was therefore deemed to furnish the most promising material for testing any new formula. This series is the so-called "singlet" series running from the first head of the band. Its structure has been studied by Kayser and Runge,[2] Jungbluth,[3] Uhler and Patterson,[4] Kilchling,[5] and others. The lines of the series have been observed from $m=4$ to $m=168$ inclusive.[6] In Fig. 2 the solid line marked A_1 represents a *smooth* curve drawn through the experimental values, in terms of $\Delta\nu$ and m,[7] the mean position of all resolved doublets being used. The actual data for the A_1 series used in this article are taken exclusively from the results of Uhler and Patterson.[8] The

[1] See Baly, *Spectroscopy*, pp. 564–567 (1912 ed.).

[2] *Abhandl. der Berliner Akademie*, 1889; also Kayser's *Handbuch*, **2**, 479, and **5**, 230.

[3] *Astrophysical Journal*, **20**, 237, 1904.

[4] *Ibid.*, **42**, 434, 1915. [5] *Zeitschrift für wiss. Photographie*, **15**, 293, 1916.

[6] This is really a series of doublets, but the two components, at the head of the series, apparently coincide in position. From this point the separation increases to a maximum of about 0.07 A, remains roughly constant for some time, and then rapidly diminishes. The first and last resolved doublet is about $m=60$ and $m=154$, respectively.

[7] In order to obtain a convenient 1-to-1 scale for plotting, frequency is given in terms of 10^9/A, instead of the usual 10^8/A. In this article frequency and first frequency-difference in 10^9/A are denoted by ν and $\Delta\nu$, respectively, while the same quantities in 10^8/A are denoted by ν' and $\Delta\nu'$.

[8] The frequencies and first frequency-differences, reduced to *vacuo*, are given by these authors for the A_1 series only. Professor Uhler has asked me in this connection to call attention to the fact that in reducing the wave-lengths to *vacuo* the index of refraction of air at 20° C. was used by mistake instead of that at 15° C. As a result, the frequency for $m=0$ is 0.124 ν' too large, while that for $m=168$ is 0.136 ν' too large. But the *relative* error thus introduced is only 0.012 ν' (= .0018 A) and therefore is negligible, as far as series relationships are concerned.

author, however, has a series of very fine exposures of the entire arc spectrum of carbon, taken with the 21-foot concave grating of the University of Wisconsin. Portions of these plates have been studied with an 8-inch Gaertner comparator, which the author was enabled to purchase through the generosity of the Rumford Fund Committee. The immediate object was to attempt to extend the various series of the 3883 band, and also to clear up

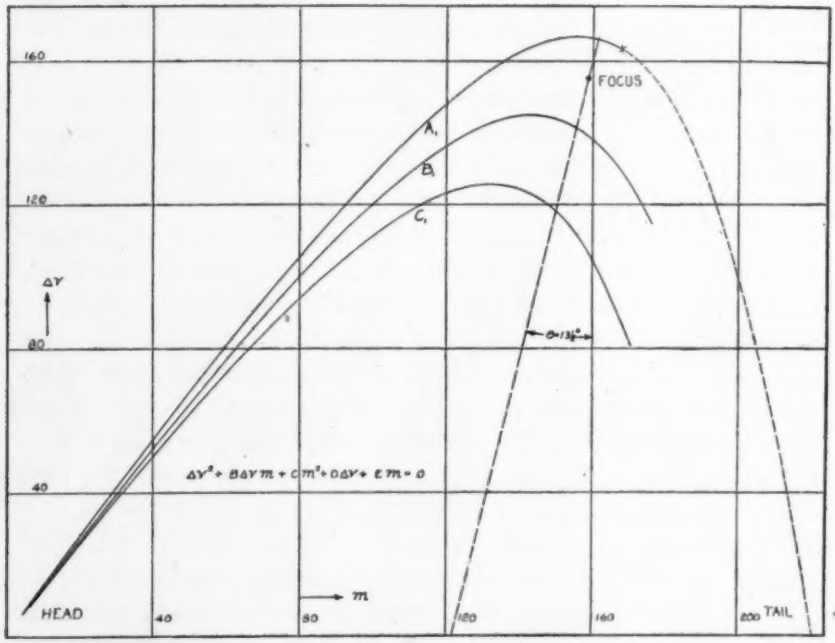


FIG. 2

certain perplexing points in the structure of the known portions. This investigation is still in progress, and the results will be presented in full in a separate paper.

It has not been possible to extend the A_1 series beyond $m=168$, the point at which all previous investigators have stopped. There is, however, no question as to the identity of the lines up to this point. From $m=160$ on, the intensity is rapidly diminishing, but at $m=168$ it is still great enough to indicate the possibility of following the series for several more lines. There is, in fact, a

series of lines which roughly forms a consistent extrapolation of the curve given in Fig. 2, but the intensities and appearances of these lines are consistent neither with each other nor with the last known A_1 lines. It must therefore be concluded that the A_1 series ends at $m=168$. This is rather unexpected, for in the case of the B_1 and C_1 series (the corresponding series of the second and third heads) the intensity falls off uniformly. The author has succeeded in extending the C_1 series several lines beyond the previous limit, and the last two or three lines are so faint as to be barely visible. On the other hand, both the A_1 and B_1 series end at about the *same* wavelength (3640 A), while the C_1 series ends 20 A previous to this point. Beyond 3640, as other investigators have noted, the band loses all appearance of regular structure, while the background is filled with a great number of seemingly unrelated lines.

When the $\Delta\nu$-m curves for the B_1 and C_1 series had been computed and plotted (as shown in Fig. 2[1]), it was immediately evident that they would furnish a far more critical test of any new formula than would the A_1 series. For, while this series stops just past the maximum $\Delta\nu$ (which occurs at $m=156.5$), the B_1 and C_1 series run very much farther. On the other hand, the A_1 series is distinctly the strongest in the band, and has the smallest "perturbations," so that the data for this series are more complete, more accurate, and more consistent than for any other series. The work on the B_1 and C_1 series has now been carried to the point where it is evident that the theory of band structure here presented applies with equal accuracy to these series. The actual results will be presented in another paper. The present contribution deals exclusively with the A_1 series, using the mean position of each doublet.

Kayser and Runge[2] concluded that this series could be represented satisfactorily by the four-constant formula

$$\nu=A+Be^{\alpha m}\sin(\beta m^2), \tag{6}$$

to which, however, they attached no importance, because of its

[1] The portion of C_1 between $m=6$ and $m=47$ is based on the author's own measurements, the series not having previously been identified over this interval. The same statement is true for the portion $m=164$ to $m=171$.

[2] *Ibid.*

complicated form. But, since the differences (O.–C.) amount to as much as 0.4 A in at least one place, the formula can scarcely be called satisfactory. The probable relative errors in the data of Uhler and Patterson are considered by the authors to be ±0.002 A for sharp lines and ±0.01 A for hazy lines. A smooth curve, such as is given in Fig. 2, shows that the average deviation of the experimental points is in practical accordance with this estimated error. (This is exclusive of the perturbations, which consist, usually, of a single line of exceptional character, displaced considerably from its expected position. The perturbations of similar series are similarly located, and all perturbations show exceptional behavior in a magnetic field.[1] They are undoubtedly destined to assume an important place in any complete theory of molecular structure.)

Kilchling,[2] in a painstaking investigation of which, unfortunately, only the first half has yet reached the writer, concludes that no three-constant formula can be used to represent this series. No attempt was made to investigate four-constant formulae, because of the great variety of possible forms. Instead he decides to use the purely empirical formula (as he himself calls it)

$$\nu = a + bm^2 - cm^4 + dm^6 - em^8, \qquad (7)$$

in which the five constants are intrinsically positive. This corresponds directly to the Kayser and Runge formula for line series

$$\nu = a + bm^{-2} + cm^{-4} + \text{etc.},$$

which the authors carried to three terms only, although several more are necessary for real accuracy.

No hint as to how accurate Kilchling found formula (7) is given in the first half of his article. It is, however, possible to draw certain general conclusions as to the applicability of any general interpolation formula, such as (7). If repeated frequency-differences are computed, each line or, more conveniently, only every fifth or tenth line being used, then, if the pth order of frequency-differences is essentially constant, the series can cer-

[1] Fortrat, *Comptes rendus*, **156**, 1459, 1913; **157**, 991, 1913; **158**, 334, 1914; Deslandres and d'Azambuja, *ibid.*, **157**, 671, 814, 1913; **158**, 153, 1914; Deslandres and v. Burson, *ibid.*, **157**, 1105, 1913; **158**, 1851, 1914.

[2] *Loc. cit.*

tainly be satisfied by the general equation of the pth degree, having $(p+1)$ arbitrary coefficients. And it *may* happen that any one or more of the coefficients, *except* that of x^p, can be equated to zero without appreciably disturbing the agreement. Thus, if the differences of the 8th order are constant, formula (7) *may* be satisfactory, although more likely it will be found necessary to include also the odd-power terms. Now, the data taken from a smooth curve for the A_1 series show that the sixth differences are nearly enough constant so that a sixth-degree (7 constant) formula would be satisfactory to 0.01 A throughout. For the C_1 series similar computations show that it would be necessary to add one or more higher powers. Kilchling thus uses, for the A_1 series, a higher power (8th) than necessary, but the dropping of the odd powers necessitates this.

The author has tested the Kilchling formula on the A_1 series and finds that the deviations (O.–C.) cannot be brought within the limits of experimental error. This statement would be true in far greater degree for the B_1 and C_1 series. The Kilchling formula gives a "slope" curve which is too straight near the origin, but does not bend rapidly enough near the point of maximum first frequency-difference.

CALCULATIONS

The shape of the curves in Fig. 2 immediately suggests some sort of a conic, while the importance of this type of curve in all dynamic problems is an argument in favor of its use as an empirical formula.

In testing the A_1 series, I therefore first tried a parabola running through the origin and having its axis parallel to the $\Delta\nu$ axis (two undetermined coefficients). This was entirely unsatisfactory. Next I tried a hyperbola through the origin, with the axis parallel to $\Delta\nu$ (three coefficients). This came very near to being satisfactory, and a considerable amount of time was spent in a vain endeavor to get a perfect agreement. Finally, I tried a hyperbola through the origin, with the axis having any slope (four arbitrary constants), and this was found *entirely satisfactory*. The equation in $\Delta\nu$ and m is therefore

$$\Delta\nu^2+B\Delta\nu\cdot m+Cm^2+D\Delta\nu+Em=0. \qquad (8)$$

For convenience the coefficient of $\Delta\nu^2$ is taken as the one to be made arbitrarily equal to unity. The fact that the curve goes through the origin eliminates the fifth constant. I wish, however, to lay stress on the fact that the absence of the constant term is not a material point. Both the A_1 and B_1 curves evidently head straight for the origin. But the C_1 curve, as recently determined, does *not*. It makes little difference in the actual calculations whether (8) or the complete equation of the second degree is used, since the elimination of the constant term between five simultaneous equations is a very simple process.

If (8) is solved explicitly for $\Delta\nu$, we have

$$\Delta\nu = -Bm/2 - D/2 \pm \sqrt{\left(\frac{B^2-4C}{4}\right)m^2 + \left(\frac{BD}{2}-E\right)m + D^2/4}. \qquad (8')$$

The branch of the hyperbola actually represented by the series corresponds to the negative sign before the radical. The best set of values of the constants for the A_1 series, in terms of the new unit of $\Delta\nu$, $(10^9/A)$, is

$$B = +\ 7.2383694 \qquad D = -2{,}101.1672$$
$$C = -13.258107 \qquad E = +2{,}929.5502$$

The broken lines in Fig. 2 give the extrapolation of the hyperbola obtained with these constants, and its axis.

No particular importance should be attached to the absolute values of the constants, since the slightest change in the shape of the hyperbola (especially near the vertex) causes a radical change in the absolute values, although but little change in the relative values. Because of this fact, it was not found expedient to use a least-square solution, since this does not in general give satisfactory results unless very approximate values of the unknowns have previously been determined. The procedure used was to pass the curve through four arbitrary points by solving the four simultaneous equations, and then to shift and reshift until the agreement was satisfactory. This is unfortunately a rather laborious process, for the fitting of the hyperbola, especially around the vertex, is a delicate operation. It is also unfortunate that the data in this region are poorer than for the lower part of the curve.

The conditions are just reversed in the case of line series, for which the procedure is perfectly straightforward.[1]

The accuracy with which the equation represents the experimental data is shown by the broken-line curve in the upper half of Fig. 3. Here the O.–C. value of each $\Delta\nu$ is plotted against the corresponding m. The center line of each three indicates the axis, while the other two lines are drawn on either side at a distance representing 0.01 A.

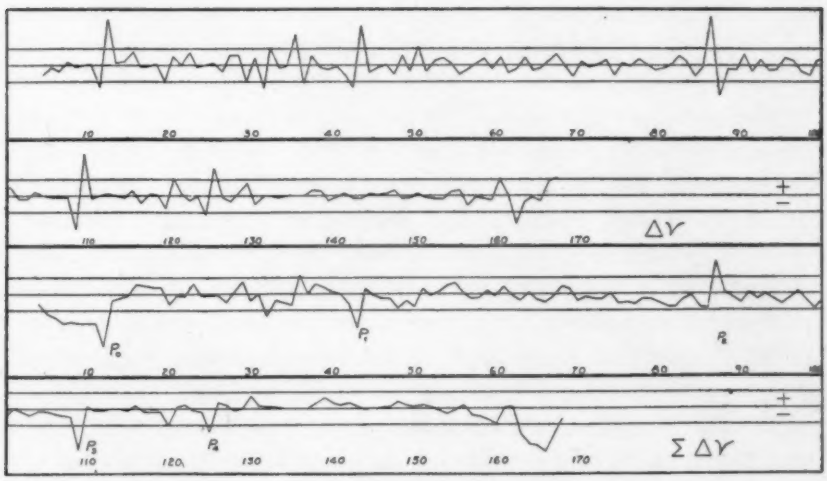


FIG. 3

It is, however, the final agreement between observed and computed frequency that is of real importance. As has been pointed out, the frequency is given by

$$\nu = \nu_0 + \Sigma\Delta\nu, \tag{9}$$

and this is *not* equal to $\int\Delta\nu$. For this particular curve I find that the difference between the summation and the integral amounts to about 0.005 A for the very first line (the $\frac{1}{2}\Delta\nu_{0.5}$ rectangle), and that this difference gradually diminishes to a value of about 0.0025 A at the next to last line and then jumps to 0.005 A for the last line (owing to the $\frac{1}{2}\Delta\nu_{n-0.5}$ rectangle). Thus, as far as fitting

[1] R. T. Birge, *Astrophysical Journal*, 32, 114, 1910.

the experimental data is concerned, the integral might equally well be used, instead of the summation.

But the integral of (8) is very complicated, having the form

$$\nu = C + am + bm^2 \pm \{(cm+d)\sqrt{em^2+fm+g} + h \sin h^{-1}(lm+k)\} \quad (10)$$

where a, b, ... k, are $f(B, C, D, E)$. I have therefore not tried to work with it, but instead have taken the successive $\Sigma\Delta\nu$. If one wishes to test at every point the agreement between observed and computed values, it is of course necessary to compute each $\Delta\nu$, and after that is done the actual frequencies follow immediately.[1]

The final agreement between observed and computed frequencies is given in the lower half of Fig. 3, the scale being the same as for the upper half. The two curves are so related that any point on the axis of the $\Sigma\Delta\nu$ curve indicates that in the $\Delta\nu$ curve the algebraic sum of the deviations, from $m=0$ up to the point in question, is zero. However, the $\Sigma\Delta\nu$ curve begins with a residual error of 0.006 A, which is not indicated on the $\Delta\nu$ curve. This arose from the particular value (ν_0) assumed for the head of the series. This value, in turn, was chosen so as to make the computed frequencies agree as well as possible with the observed frequencies for the first twenty or thirty known lines.

As Fig. 3 shows, the observed and computed frequencies agree to within 0.01 A throughout, with the exception of the perturbations (marked P_1 . . . . P_4) and slight irregularities at the two ends. In fact, the average residual (O.–C.) for the entire 164 known lines is 0.005 A, while, if the perturbations and irregularities mentioned are excluded, the average is only 0.003 A. This value is clearly within the limits of experimental error and constitutes a greater accuracy than has ever before been attained for any long band series. The only line-series formulae which have given better results are the three-constant formula with which W. E. Curtis[2] has fitted the hydrogen Balmer series and the six-constant formula used by Nicholson[3] for the helium series. The four-

[1] One method of computing any summation is that of finite differences. But the high order of differences required in this case, as shown in the discussion of the Kilchling formula, renders the method useless for extrapolation. It can, however, be used profitably for interpolation.

[2] *Proc. Roy. Soc.* (A), **90**, 605, 1914. [3] *Ibid.*, (A), **91**, 255, 1915.

constant formulae of Ritz and Hicks give deviations up to at least 0.1 A for all series save those just mentioned. It is therefore not unreasonable to expect that a formula with at least five constants, such as (9), will be necessary for the longest and most complex band series, when three to six constants are needed for the simplest line series.

The reasons for excluding the initial and final irregularities in computing the final accuracy are as follows:

1. The lines from $m=160$ to the end are broad and hazy. The experimental error is great enough to cover that shown in the curves of Fig. 3. I have tested this by taking several sets of readings on these lines and by obtaining error-curves entirely unlike those in Fig. 3.

2. The point marked P_0 at $m=12$ is not a true perturbation, and the errors to the left of this point are only apparent. A critical examination of the first three heads of this band has shown that they are essentially similar, and that each has the following structure. The head is composed of a *double line*, which can actually be detected as double on some of the negatives, and this doublet constitutes the *first member* of the "doublet" series (which has practically constant separation). The singlet series begins *in coincidence* with one component of the doublet series (the more refrangible, in the case of A_1 and C_1; the other, for B_1). The singlet series gradually leaves the doublet, drawing away in such a direction as not to cross the other component of the doublet.

The first line at which Uhler and Patterson resolve the A_1-A_2 coincidence (A_2-A_2 signifies the doublet series) is $m=13$. Therefore, from $m=12$ to $m=4$, the reading given is really the mean position of A_2-A_2, and not the true position of A_1. The $\Sigma\Delta\nu$ curve in Fig. 3 should therefore show an abrupt drop to about -0.03 A at $m=12$, as it does, and this apparent error should decrease linearly to zero at $m=0$.

The value of ν_0 that it was found necessary to use in (9) is $25{,}743.79\,\nu'$. This is $0.33\,\nu'$ ($=0.05$ A) higher than Uhler's measured value for the head of the band. But this difference is in perfect accordance with the constitution of the head. For the separation of the doublet series, at the head, is 0.05 A, and the A_1 series begins in coincidence with the *inside* component.

As has been mentioned, the actual constants given for equation (8) apply to the mean position of all resolved doublets of the A_1 series. The peculiar variation of the separation in these "singlet" series, from zero to a maximum and then back to zero, is, however, consistent with the formula just proposed, so that it would be quite possible to compute for each component a separate set of constants, yielding equally satisfactory results.

DISCUSSION OF RESULTS

The main objections (practical and theoretical) to this theory of band structure seem to the author to be:

1. The equations (8) and (9) involve a simple and definite relation between first frequency-difference and m, but *not* between frequency and m.

This may not be a true objection, for it is possible that the theoretical physicist is just as interested in a property of $\Delta\nu$ as he is in one of ν. In fact, Hasenöhrl,[1] using quantum theory, derives a simple expression for $\Delta\nu$ which apparently can be used to explain the structure of band series.

2. Equation (9) involves a summation, instead of an integral, and the integral which approximates it in value is too complex to be of any practical utility.

The summation is necessary from the very nature of the case, and, as is only too well known, the summation and difference seem destined to assume a very prominent place in the mathematical physics of the future. I have not thus far found any really satisfactory approximation for the summation. The first natural suggestion is to expand the radical of (8′) in ascending powers of m and integrate term by term. The series so obtained is convergent throughout the experimental range, but the rate of convergence is so slow that some forty or fifty terms would be necessary for the requisite accuracy.

Perhaps the best *rapidly* converging series which will approximately represent the hyperbola is

$$\Delta\nu = am + bm^3 + cm^5 + dm^7, \qquad (11)$$

[1] *Physikalische Zeitschrift*, **12**, 931, 1911.

for which both the integral and the summation will have the functional form of Kilchling's equation (7). The coefficients in the two cases, however, will be slightly different.

The accuracy with which the hyperbola fits the experimental data seems to indicate that we have here a true relation between $\Delta\nu$ and m, and in this respect the case is unique. For all other spectral series formulae have in them an unknown $f(m)$, which is taken usually as a semiconvergent series, and enough terms retained to give the required accuracy. Assuming, then, that the hyperbola *is* a true relation, we have the following structure for band series:

The series starts at $m=0$ (the head) and the first frequency-differences initially increase almost linearly. The values of $\Delta\nu$, however, finally reach a maximum and then decrease to zero. The point at which $\Delta\nu$ is again zero constitutes the tail of the series. It has the same structure as the head, hence a weak band might be the tail of a strong band degraded in the opposite direction.

In the case of all known band series the intensity has dropped to zero (or the series has been concealed by another band) before the tail has been reached, and for nearly all series this is true even before the point of maximum $\Delta\nu$ has been reached. In the case of the A_1 series the tail is given by $m=221$ $(=-E/C)$ and should appear at 3567.2 A. This position is in the 3590 band, and so the tail, if present, could not be observed. In the case of the C_1 series the tail is given (approximate calculation) by $m=191$, so that the last observed line ($m=171$) is only *twenty lines* from the tail and at a distance of less than *twelve angstroms*. But no one of the observed tails is near the predicted position, and there seems to be no apparent reason why the intensity, after having dropped to zero, should increase again in the vicinity of the tail.

The values of $\Delta\nu$ and m for the vertex of the hyperbola and the slope of the axis (13°.5 for A_1) can have no theoretical significance, since they depend upon the particular units chosen for $\Delta\nu$. For, if $\Delta\nu'$ $(=k\Delta\nu)$ is substituted in (8), a new hyperbola with a new vertex and new axis is obtained. The only object in stating such an obvious fact is to call attention to the following error which the author has already made.

A preliminary examination of the A_1, B_1, and C_1 curves seemed to indicate that the three hyperbolae were identical in size and shape and differed only in position. But the exact results for A_1 showed that this was not the case. It then appeared that the axes of the three hyperbolae were parallel, and this hypothesis was mentioned at the April 1917 meeting of the American Physical Society, the implication being that such a result might indicate some important relation between different series. But the simplest algebra shows that, if the axes are parallel for one $\Delta\nu$ unit, they will not be parallel for another, *unless* the B and C constants of (8) are the same for each series. And, if they are the same, the hyperbolae will be identical in size and shape—a condition known to be contrary to the facts. Therefore the parallelism of the axes, if true, is purely fortuitous and without meaning.

CONCLUSIONS

The only band series which can afford a real test for the hyperbolic formula (8) are those in which the first frequency-difference actually attains a maximum value. The ordinary band series forms only a short segment of the hyperbola and so without doubt can be satisfied equally well by many other four- or five-constant formulae. But the only band series, so far as the author is aware, which attain a maximum $\Delta\nu$ are those of the CN bands, and several series in the bands of phosphorus.[1] These latter are only fifty and sixty terms long; they have been measured with far less accuracy than the CN series; and, like the A_1 series, they run but a few terms beyond the maximum $\Delta\nu$. For these three reasons they would furnish but inferior material for a test.

It therefore seems that formulae (8) and (9) will be found satisfactory for *all* ordinary band series (Konen's bands of the "first type"), *with the single exception* of certain series in the 2370 band of air, for which Deslandres[2] states that the deviations from his law are in the direction *opposite* to that indicated by the hyperbola. These series therefore require special investigation.

[1] Geuter, *Zeitschrift für wiss. Photographie*, **5**, 1, 1907.

[2] Deslandres and A. Kannapell, *Comptes rendus*, **139**, 584, 1904.

SUMMARY

1. No satisfactory formula for long band series has thus far been proposed.

2. In the case of the main (A_1) series of the 3883 CN band—a series of 164 known lines—it has been found by the author that the relation between first frequency-difference ($\Delta\nu$) and a variable m is that of a hyperbola. The hyperbola cuts the origin, but does not lie parallel to the co-ordinate axes, so that it is given by a four-constant equation.

3. The actual frequencies are then given by the five-constant formula, $\nu_0+\Sigma\Delta\nu$, and the agreement between observed and computed values is of such accuracy as to indicate that the hyperbola represents a true relation and one, it is hoped, which will readily lend itself to a possible theory of molecular structure.

4. The $\Sigma\Delta\nu$ cannot conveniently be replaced by the $\int\Delta\nu$, and the formula has some troublesome features, but these are shown, in the light of the experimental facts, to be not unexpected.

5. According to this new formula, a band series should end in a tail having the same structure as the head of any band series, in contrast to Thiele's tail, the structure of which should be that of the head of a line series.

6. No series has been followed to its tail, and in the case of three series of the 3883 band there are no observable tails at the predicted positions, although in one case the series can be followed to within 12 A of the hypothetical tail. The intensity relationships, however, do not indicate that the tail should actually appear.

7. But few band series show sufficient deviations from the simple Deslandres' law to constitute a rigid test for the proposed hyperbolic formula, while in only one known band are the deviations of a nature contrary to that predicted by this formula.

8. The numerical results for the A_1 series only are presented in this paper. Those for the B_1 and C_1 series will be published later.

SYRACUSE UNIVERSITY
DEPARTMENT OF PHYSICS
May 26, 1917

A METHOD OF INVESTIGATING THE STARK EFFECT FOR METALS, WITH RESULTS FOR CHROMIUM[1]

By J. A. ANDERSON

The object of the present work was to find a method suitable for the study of the Stark effect for metals having a relatively high melting-point. The spectra of such metals as iron, nickel, chromium, vanadium, and titanium are of special importance in solar work, and if some of their lines show an effect due to an electric field it would at once become possible to determine whether electric fields exist in the solar atmosphere. In previous work on the Stark effect some of the lines of these metals have been observed, but in no case has an electric effect been found for them. Generally the source used has been of feeble intensity as compared to an arc or spark, necessitating the use of low dispersion, and hence the first problem which had to be solved in the present investigation was that of getting a source sufficiently bright to be studied with a spectrograph of moderate dispersion.

APPARATUS

The apparatus shown in Fig. 1 has given general satisfaction; most of the lines of the metals enumerated above can be recorded in exposures of from 10 to 30 minutes to a scale of about 4 A per millimeter, using a spectrograph such as that described in this paper.

A bell-jar, about 15 cm in diameter and about 30 cm high, stands on a glass plate in which there are two openings. The connection to the pump is sealed into one of these, while a larger glass tube, having a platinum electrode at its closed end, is sealed into the other. This larger glass tube is about 2.5 cm in outside diameter for about one-half its length, then tapers to about 1.5 cm in order to form a seat for the ground joint with the long silica tube *D*. *D* is about 20 cm long and 1.2 cm in outside diameter, and carries at its upper end the short silica tube *C*, to which it is

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 134.

fitted by a ground joint. The cathode is in two parts—*B*, made of iron, and *b*, of the metal which is to be studied—and it rests on top of the tube *D*. *C* has a vertical slot, about 1 mm in width, which extends down to the level of the top surface of *b*, the cathode proper. The tube *C* must be ground truly cylindrical inside and must be of a diameter about 0.1 mm larger than the cathode *b*. The necessity for this will be discussed below.

A fine platinum wire, 33 B. & S. gauge, is attached to *B* by means of a screw and carries at its lower end a short brass cylinder which makes contact with the platinum electrode in the larger glass tube. A mercury contact at this point was tried first, but the mercury vapor gave rise to so much trouble that it became necessary to resort to this form of metallic contact, which has worked admirably.

The anode is a massive disk of aluminium on the end of an aluminium rod. A thick platinum wire, fastened to the rod, is sealed into a glass tube, as indicated at the top of the diagram. The knob at the top of the bell-jar has a hole drilled through it, into which the glass tube is fitted by grinding; it is then sealed in with water-glass.

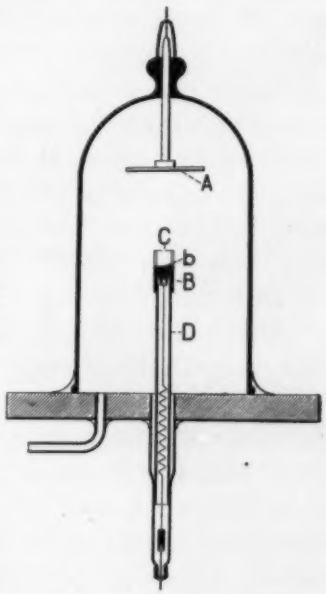


FIG. 1.—Diagram of apparatus. *A*, Anode; *Bb*, Cathode; *C*, Silica tube with slot for observing; *D*, Silica tube.

Direct current to produce the discharge is supplied by a set of high-potential generators connected in series. Each generator gives approximately 800 volts, and for most of the work 8 generators were used. Two spectrograms were obtained with the use of 16 generators, but so much difficulty was experienced in keeping the vessel operating satisfactorily at this high voltage that it was discontinued until some future time, when it will be tried again. A resistance consisting of distilled water with platinum electrodes was always used in series with the tube.

The resistance varied somewhat, but in general it was kept at about 32,000 ohms.

A high-vacuum oil pump is used to exhaust the bell-jar, the pressure employed being such that the Crookes' dark-space has a length of from 4 to 6 mm. At first hydrogen was used as residual gas, but later it was found that spectra of the metals are brighter in air or oxygen, and hence in the major part of the work commercial oxygen was employed. The gas was passed through concentrated sulphuric acid to remove most of the moisture, but, since no drying tube was connected to the vessel directly, there were always traces of water-vapor present. This was found to be a real advantage, as it gave the hydrogen lines with an intensity which was very suitable for determining the field-strength.

PHENOMENA OF THE DISCHARGE

With the use of 8 generators (about 6400 volts), a resistance of about 32,000 ohms in series, and pressure adjusted to give a dark-space of about 4 mm, the initial current, if one of the metals already mentioned is used as a cathode, will be about 100 milliamperes. With manganese the current is usually a little higher, while with magnesium it is about 200 milliamperes, the bell-jar acting as if it had practically no resistance. During the first minute the current usually decreases rapidly, and in about two minutes it becomes steady at approximately 40 milliamperes. In the meantime the cathode *b* has become almost white hot, its temperature being now from 800° to 1000° C. The cathode glow, Crookes' dark-space, and most of the negative glow are confined within the tube *C*, while a soft glow extends with decreasing intensity from the top of *C* up to the anode or even beyond it. On the anode may be a single bright point, or there may be nothing whatever visible. After the current has run about ten minutes, all of the tube *C*, excepting a few millimeters near the top, will be red hot, as will also the cathode *B* and the upper end of the tube *D*. The bell-jar will have warmed up considerably, too, and usually the middle portion of it will be too hot to be touched by hand, while the upper and lower parts will be relatively cool.

In order that the heating of the vessel may be equalized as much as possible, a hollow cylinder made of sheet brass is placed around the cathode. This cylinder, which is about 9 cm in diameter and 18 cm long, rests on the base-plate and reaches to within about 2.5 cm of the anode *A*. A suitable opening opposite the slot in *C* allows the light to pass out to the spectrograph, while the observer can see the cathode by looking obliquely down between *A* and the top of the cylinder. With this arrangement the heating of the bell-jar is much more uniform and also considerably slower, so that an uninterrupted run of from 30 to 60 minutes becomes possible. Since glass expands considerably with a rise in temperature, while silica expands very little, it is obvious that the ground joint between silica and glass must be located where there is little or no change in temperature. Hence this joint is placed well below the base-plate and is so arranged that it can easily be surrounded by a water bath if necessary.

If the cathode does not come in contact with the tube *C*, and if at the same time is not separated from it by too large a space, the discharge takes place only from the upper surface of *b*. If the separation between *b* and *C* is too large, the discharge tends to start from *B*, or even from the wire below *B*, and passing in a thin stream between *b* and *C* produces intense local heating, which soon causes little arcs to form. If, on the other hand, *b* and *C* are in contact, then, when the discharge has passed long enough to heat *C* to redness, *C* itself begins to act as a cathode, and no result can be obtained. Great care is therefore necessary in adjusting the cathode and the tube *C* so there shall be no contact, even after *b* has become white hot.

The only fault the writer has to find with the present design is that after several runs the inner surface of *C* becomes so thickly coated with the metal of the cathode that a contact or short-circuit is unavoidable. The only remedy is to let in air, remove the bell-jar, and clean the tube, which requires only ten minutes of actual manipulation, but really involves two or three days' delay in the experimental work, as will be more fully explained in connection with the results for chromium.

With several days' continued use the upper surface of the cathode *b* becomes hollowed out into a low, nearly flat-bottomed

dish, with a rim from 0.5 to 1 mm thick, which is not affected. In order to be able to observe the discharge at the surface of the cathode from the side, it is of course necessary to remove this rim occasionally. In the present work this has been always done when the depth of the dish reached about 0.5 mm.

As is well known, the electric field is by no means uniform in the Crookes' dark-space, but has its greatest intensity at the surface of the cathode, decreasing as the distance from the cathode increases, until it becomes zero at the edge of the negative glow. The rate of decrease of the field with the distance from the cathode—or, better, the rate of increase with the distance measured from the negative glow—is a matter of considerable importance, and it might be expected that this rate would be different for different apparatus.

In the ordinary Lo Surdo type of tube, which has a diameter of from 1 to 2 mm, Takamine and Yoshida[1] have found that the field-strength varies approximately as the square of the distance from the negative glow, so that the components of the hydrogen lines become approximately parabolic curves, the vertices being in the negative glow. In the present apparatus the field-strength varies almost linearly with the distance, as can be seen from Plate X*a* which shows Hγ, the *n*-components above, the *p*-components below. The components are nearly straight lines. This is a very fortunate circumstance, since one can tell at a glance, when examining lines of other elements, whether the displacement varies linearly with the field-strength or not, merely by seeing whether the components are straight lines or curves.

THE SPECTROGRAPH

This is a plane-grating, two-lens type, with the entire mounting constructed of wood in order that changes might easily be made should experience show that the original design could be improved.

The collimator lens is a Voigtländer portrait objective of 38 cm focus and 10 cm aperture; the camera lens is a Cooke astrographic of $F/4.5$ aperture and 45 cm focus. The grating is a 4-inch Rowland, with 14,438 lines per inch, having a rather bright third

[1] *Memoirs of the College of Science* (Kyoto Imperial University), **11**, No. 2, pp. 137–146, 1917.

order. It is mounted in such a manner that its normal always makes an angle of 70° with the axis of the camera lens. The collimator is fixed in the wooden housing of the spectrograph, in order that the discharge vessel and projection system may remain in a fixed position. The camera and grating can rotate through an angle of about 30° around a vertical axis through the center of the grating surface, thus bringing the different portions of the spectrum on the photographic plate.

As set up, the third order can be covered from about λ 3000 to λ 5000, and the second order from λ 4500 to λ 7500. By giving the collimator another fixed position, of course it would be possible to observe all of the first order or all of the fourth order instead of the regions chosen. An advantage of this form of mounting is that the dispersion at any given point of the photographic plate is always the same, no matter what region of the spectrum happens to fall there. Hence, the dispersion for all points of the plate having been determined, either by a photograph of a known spectrum or by calculation, a scale in angstrom units may be ruled which will fit all plates closely enough for purposes of identification.

The plates used measure about 6×10 cm, and the plate-holder is movable in a vertical direction, so as to allow several exposures on the same plate. In the third order about 400 A are covered with one exposure, the dispersion varying from about 5.2 A per mm at the violet end to about 3.0 A per mm at the red end. In the second order the dispersion is, of course, just two-thirds of that in the third order.

With short exposures pairs of lines having a separation of 0.1 A can as a rule be resolved. With the longer exposures, which are subject to disturbances from temperature-changes and vibration, the practical limit of resolution is about 0.15–0.20 A. The accuracy of wave-length determinations has been found to be easily of the order of 0.01 A.

An image of the slot in *C*, Fig. 1, is projected with a magnification of one-third on the slit by using two achromatic lenses, each of 20 cm focus and 5 cm diameter. Before falling on the projecting lenses the light passes through a double-image prism, so that two images, one immediately above the other, appear on the slit, the

vibrations in the upper image being vertical, or parallel to the slot, in the silica tube *C*.

An exposure usually requires from 20 to 30 minutes, but the vessel has then become so hot that from one to one and a half hours are required to cool it before another exposure can be made. (Of course in future work this waste of time will be eliminated by the construction of a vessel which can be cooled artificially, thus making it possible to operate without stopping.)

About one hundred photographs have been taken up to the present time, nearly one-half of which were made with hydrogen as residual gas. These include spectra of iron, chromium, nickel, vanadium, and calcium. Only chromium has been at all carefully studied, however, and the present paper will be limited to a preliminary report on those lines in the spectrum of chromium which show an electric decomposition.

RESULTS FOR CHROMIUM

The chromium spectrum has been photographed from λ 3670 to λ 5410. In the earlier spectrograms very little of interest from the viewpoint of a Stark effect was observed; the normal arc lines were always present, but of the sensitive lines only faint traces of the groups at λλ 4098, 4111, and 4129 could be seen. These showed, however, a suspicion of an electric effect, and hence for some days this region was photographed repeatedly in the hope of bringing out the effect more clearly. One spectrogram happened to show these groups with an intensity something like one hundred times as great as the others, without, however, any marked increase in intensity of the normal arc lines such as λ 4254. Viewed with a small direct-vision pocket spectroscope, the triplet near λ 5200 is always quite intense, but, when the spectrogram just mentioned, No. 55, was made, it was noted that three lines on the red side of the green triplet had an unusually high intensity. These "lines" were later found to be the groups at λλ 5275, 5298, and 5329, and their appearance was always used to indicate the character of the discharge.

Considerable difficulty was encountered in obtaining another spectrogram like No. 55. The record showed that in making this

exposure the current was 30 milliamperes; the length of the dark-space, 5 mm; the voltage, approximately 6000; the exposure time, 20 minutes; and the residual gas, air. These conditions were exactly reproduced time after time, and still only mere traces of the sensitive lines were obtained. In these trials the apparatus was as a rule taken down and thoroughly cleaned between exposures. Since this did not lead anywhere, a number of successive runs were made without taking down the apparatus and without letting in any more gas than just necessary to keep the experimental conditions correct. Then it was found that the sensitive condition as a rule developed quite suddenly after about half a dozen successive runs, and, once formed, seemed to persist in successive exposures, until it became necessary to clean the apparatus as a result of a contact between the cathode and the metal deposited in the tube *C*.

Just what this sensitive condition of the cathode is, the writer is not prepared to say; it can be destroyed by letting in air to a pressure of about 1 cm of mercury or more, it seems to form equally well in air and in commercial oxygen, and a small amount of hydrogen or water-vapor does not seem to prevent it; whether it will form in pure hydrogen is not definitely known, but indications are that it will. An exactly analogous condition has been found for iron and nickel, and it would not be surprising if it should turn out to be quite general.

No doubt it is analogous to the condition of the cathode, which is familiar to those who have had experience with the cathode sputtering of such metals as aluminium, chromium, or even nickel. The discharge may be run for a considerable time without any deposition to speak of taking place. Then suddenly the metal begins to come down, and in a short time a good coating is produced. In the present work it has been observed that the deposition on the inner walls of the tube *C* is much more rapid when the sensitive condition exists than without it; also that the upper surface of the cathode is hollowed out quite rapidly under the same circumstances. The appearance of the dark-space and of the cathode itself is also very different. Without the sensitive condition and with a current of, say, 30 milliamperes, the cathode, as seen from above, is of a purplish-red color, and the dark-space is really very dark, while,

with the sensitive condition, the cathode appears greenish-white and the dark-space is bright green and really more luminous, at least near the cathode, than the negative glow.

The lines given in the table were all recorded when the cathode was in the sensitive condition, and no doubt many more could have been recorded with longer exposures; those actually used ranged from a few minutes in the ultra-violet up to about 40 minutes in the region λ 4800–λ 5100. Most of the lines are not to be found in published tables for chromium, hence the question naturally arises: Are they really chromium lines or are they due to some impurity? The writer believes them to be due to chromium, for the following reasons:

1. The stronger lines, such as the groups at λλ 4098, 4111, 4129, 5275, 5298, and 5329, appear regularly as hazy lines in the chromium arc in air and as groups of fine lines in the vacuum arc and vacuum furnace.

2. The weaker lines recorded in the table always appear when the stronger lines are present with sufficient intensity, and do not appear when the latter are weak or absent.

3. None of these lines has been observed when any other metal was used as a cathode, although otherwise the conditions were the same.

This does not exclude the possibility that the lines may be due to some compound of chromium and nitrogen or oxygen; if this be so, then, since all come and go together, it follows that the strong lines λ 4129 and λ 5329, which appear regularly in the ordinary arc, as well as a number of other lines, such as λ 4475.50, which show with great intensity on these spectrograms, must also be due to the same compound. Now, the lines of which λ 4475.50 is an example all appear in the chromium arc as regular, though rather weak, lines; in the present investigation they appear unduly strengthened, but they show no electric decomposition.

The spectrum given by the arrangement used is somewhat different from the arc, spark, or furnace spectrum of chromium. A number of arc lines do not appear at all or appear with a relatively very small intensity; others, again, seem to appear with about the intensity to be expected; while still others are stronger than might

TABLE I

Affected Chromium Lines

λ	Int.	σ-component			π-component		
		Δλ	Int.	Rate of Displacement	Δλ	Int.	Rate of Displacement
3719.12	2	−0.76	.....	=	.....	.....	.....
3720.74	5	+0.48	.....	=	Complex	.....	.....
3722.83	2	+1.07	.....	+	.....	.....	.....
3730.09	2	−0.96	.....	=	.....	.....	.....
3731.85	5	+0.40	.....	−	Complex	.....	.....
3735.13	2	+0.91	.....	=	.....	.....	.....
3744.73	1	−0.29	.....	=	.....	.....	.....
3747.41	5	+0.32	.....	=	−0.13*	.....	=
3750.14	1	+0.63	.....	=	.....	.....	.....
3862.07	2	−0.42	.....	−	−0.42	.....	−
3863.11	1	+0.29	.....	?	+0.29	.....	?
3865.02	4	+0.25	.....	=	+0.18	.....	=
3870.28	4	+0.11	.....	?	+0.04	.....	?
3896.14	1	−1.54	.....	+	.....	.....	.....
3926.20	1	−0.16	.....	=	−0.16	.....	=
3929.87	1	+0.06	.....	?	+0.06	.....	?
4011.96	4	−0.55	.....	=	Complex	.....	.....
4013.26	4	−0.56	.....	=	0.00	.....	.....
					−0.44	.....	=
4013.99	3	−0.54	.....	=	Complex	.....	.....
4096.94	1	.....	.....	.....	−0.66	1	=
4097.41	1	.....	.....	.....	+0.21	2	=
4097.80	3	−0.68	8	+	.....	.....	.....
4098.11	4	.....	.....	.....	−0.18†	7	−
					+0.19	7	+
4098.31	4	+0.84	8	=	+0.84	2	−
4103.40	1	+0.06	1	?	+0.06	1	?
4110.75	1	−0.78	10	=	.....	.....	.....
4111.02	5	.....	.....	.....	+0.23	10	=
4111.49	4	+0.59	10	=	−0.69	2	+
4111.82	3	.....	.....	.....	−0.21	2	=
					+0.11	2	+
4112.18	1	.....	.....	.....	+0.58	2	=
4117.08	2	+0.13	.....	?	+0.13	.....	?
4128.34	1	−0.47	6	+	−0.51	2	+
4129.36	7	+0.39	8	+	−0.06	5	=
					+0.06	4	=
					+0.20	4	=
					+0.33	4	=
					+0.46	3	=
4130.13	3	+0.26	6	=	+0.26	2	−
4130.79	1	+0.40	.....	?	+0.40	.....	?
4136.25	1	+0.08	.....	?	+0.08	.....	?
4214.89	3	−0.13	.....	=	Complex	.....	.....
4215.20	3	−0.20	.....	=	Complex	.....	.....
4216.14	3	−0.13	.....	=	Complex	.....	.....

* Other weak π-components are present.

† This π-component is peculiar, as the displacement is constant throughout the entire dark-space.

TABLE I (*Continued*)

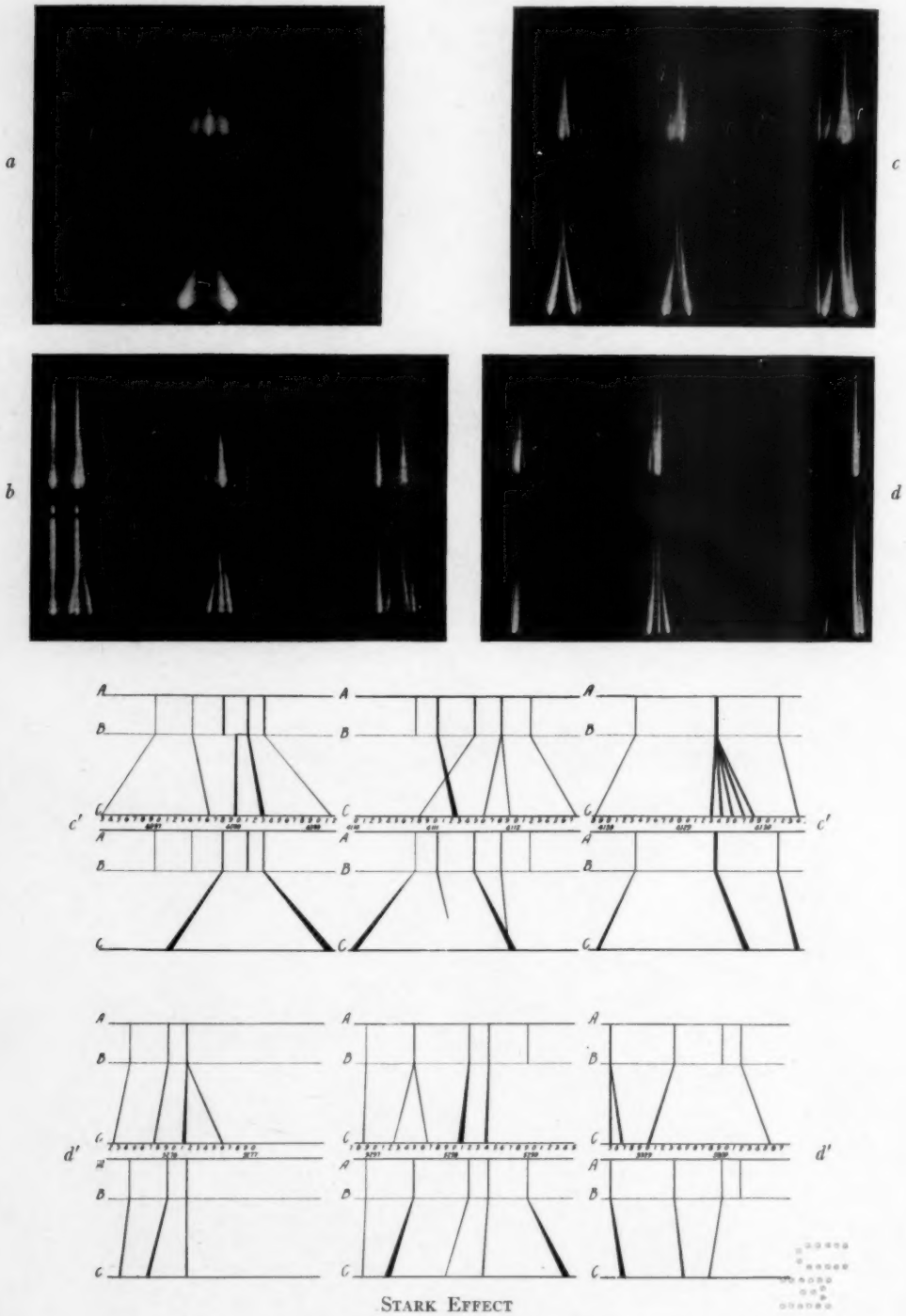
λ	Int.	*p*-component			*n*-component		
		Δλ	Int.	Rate of Displacement	Δλ	Int.	Rate of Displacement
4216.45	3	−0.15	.....	=	Complex	.....	.....
4309.05	2	+0.71	.....	+	+0.71	.....	+
4353.97	3	+0.86	.....	=	+0.86	.....	=
4389.37	2	−0.11	.....	?	−0.11	.....	?
4422.68	3	Complex	.....	.....	Complex	.....	.....
4423.15	2	Complex	.....	.....	Complex	.....	.....
4423.44	1	Complex	.....	.....	Complex	.....	.....
4453.96	2h	+0.79	.....	=	−0.43‡	.....	=
4472.03	2	+0.65	.....	=	Complex	.....	.....
4490.86	1	−0.32	.....	=	.....	.....	.....
4494.92	2	+0.54	.....	=	−0.25	.....	=
4649.59	3	+0.18	.....	−	§	.....	.....
4900.81	1	+0.30	.....	=	.....	.....	.....
4948.25	2	+0.34	.....	=	Complex	.....	.....
4983.84	1	+0.54	.....	=	Complex‖	.....	.....
4984.97	1	+0.30	.....	=	Complex	.....	.....
5005.81	2	−1.41	4	=	−0.43	3	=
		+0.53	1	=	+0.31	3	=
		+1.50	3	=	.....	.....	.....
5027.74	2	−1.60	3	=	.....	.....	.....
		+0.06	2	=	+0.21	4	=
		+0.41	2	=	.....	.....	.....
5028.37	1	+0.92	3	=	.....	.....	.....
5055.69	2	−2.19	1	=	+0.03	2	=
		+0.77	2	=	+0.54	2	=
5056.93	1	+0.42	.....	=	.....	.....	.....
5275.44	1	−0.12	2	=	−0.20	2	=
5275.92	2	−0.24	6	+	−0.18	3	+
5276.16	2	0.00	3	.....	−0.03	6	?
					+0.45	3	=
5280.81	1	+	.....	.....	.....	.....	.....
5296.88	1	−0.04	2	?	−0.03	4	?
5297.50	3	−0.34	10	=	−0.27	1	?
					+0.17	2	?
5298.18	2	−0.28	1	?	−0.12	8	−
5298.43	2	−0.06	3	?	−0.03	5	?
5298.92	1	+0.49	8	+	.....	.....	.....
5324.19	1	+0.43	3	?	.....	.....	.....
5328.52	4	+0.16	10	+	0.00	5	.....
					+0.14	5	+
5329.34	1	+0.11	4	+	−0.32	4	+
5329.94	1	−0.16	2	?	.....	.....	.....
5330.18	1	.....	.....	.....	+0.38	1	=

‡ There is also an *n*-component to the red.

§ This *p*-component is peculiar, as it seems to run without displacement from the negative glow about half-way through the dark-space, where it stops, but a line appears at this point immediately on the red side which continues without further displacement to the surface of the cathode. Breaks of a similar nature have been observed in the *n*-components of λλ 4097.80, 4111.82, and 4128.34, and the *p*-component of λ 4129.36. Some of these may show in the reproductions.

‖ There is also a *p*-component to the violet.

PLATE X



STARK EFFECT

In each illustration the *n*-components are above, the *p*-components below

a, Hγ; *b*, Chromium groups λλ 5005, 5027, 5055;
c, Chromium groups at λλ 4098, 4111, 4129;
d, Chromium groups at λλ 5276, 5298, 5329;
c', *d'*, drawings representing groups in *c* and *d*,
A–*B* being the negative glow (field-strength = 0),
C the surface of the cathode (field-strength = 12,000 volts per cm)

be expected, the group including λ 4475.50 and the stronger lines given in the table being merely those which are strengthened most. A careful study of the general spectrum given with the present arrangement is of course in order, but until a considerable number of elements have been studied in this way, so that conclusions of general validity can be drawn, it would not add materially to our knowledge, and hence a discussion of those lines of chromium which do not show any electric decomposition will not be undertaken in the present communication.

In the table the first column gives the wave-lengths as determined from the photographs, using as standards the unaffected chromium lines which appear with what is considered normal intensity; the second column gives the relative intensities on a scale of 1–10; the third column contains the displacements of the *p*-components reduced to a field-strength of 12,000 volts per centimeter; the fourth column, the intensity of the components in the stronger part of the field; while the fifth column indicates how the displacement varies with the field-strength: a plus sign (+) signifies that the displacement is proportional to a higher power of the field-strength than the first; an equality sign (=), that it is roughly as the first power; while a minus sign (−) indicates that it varies slower than the first power of the electric intensity. In a few cases where the displacement is very small, or the displaced component happens to overlap a neighboring nitrogen band-line, making measurement difficult, a question mark is placed in this column. The sixth, seventh, and eighth columns give similar data for the *n*-components.

Plate X*c* shows the groups at λλ 4098, 4111, and 4129; X*a*, the groups at λλ 5005, 5027, and 5055, while the groups λλ 5275, 5298, and 5329 are given in X*d*. The *n*-components are above, the *p*-components below, in all cases. A drawing made from the results of the measures of the groups reproduced in *c* and *d* appears in the lower half of the plate.

It will be noticed that to the violet of λ 3900 are seven lines showing *p*-components only—in other words, lines which are apparently plane-polarized in the electric field. They are all weak lines, and hence it is possible that, if the *n*-components belonging

to them are complex, they might fail to be registered on account of their very low intensity; and indeed the stronger lines in this region appear to have complex *n*-components. There are, however, several clear cases of lines with only one kind of component—for example, λλ 4097.80 and 4110.75—which have only *p*-components, and λλ 4096.94, 4097.41, 4098.11, 4111.02, 4111.82, and 4112.18, which have only *n*-components.

The lines λλ 4098.11 and 4111.02 are especially interesting; referring to Plate X we notice that the *p*-component of λ 4098.11 can be traced for only a very small fraction of the distance through the dark-space, while λ 4111.01 can be traced faintly nearly all the way across to the surface of the cathode, although it is clear that the intensity diminishes progressively as we pass down through the dark-space. This shows that in a very weak field both these lines are almost completely unpolarized; as the field increases, they gradually become plane-polarized, λ 4098.11 reaching complete polarization in a much weaker field than λ 4111.02.

A few of the lines belonging to the strong groups near λ 4100 and λ 5300 appear in the solar spectrum, and λ 5297.50 and λ 5329.34 seem well suited for an investigation of possible electric fields in the solar atmosphere.

SUMMARY

1. An apparatus suitable for investigating the Stark effect for metals having a relatively high melting-point is described.

2. With this apparatus the relation between field-intensity and distance from the negative glow is approximately linear.

3. The condition of the cathode is of vital importance.

4. A preliminary survey of the spectrum of chromium from λ 3670 to λ 5410 has been completed and a total of 74 affected lines recorded.

MOUNT WILSON SOLAR OBSERVATORY
May 1917

A DETERMINATION OF THE GALACTIC CONDENSATION FROM CERTAIN ZONES OF THE *ASTROGRAPHIC CATALOGUE*[1]

By FREDERICK H. SEARES

In a series of articles recently appearing in the *Monthly Notices*[2] Professor Turner has brought together much valuable material relating to the numbers of stars observed on the photographs for several zones of the *Astrographic Catalogue*. It was his purpose to show that important conclusions may be derived simply by counting the stars within each interval of brightness, even when the scale of luminosity is arbitrary. The result of such counts he has discussed mainly from the standpoint of systematic variations in the distribution of the stars with respect to the Galaxy; but they are perhaps equally important for a determination of the galactic condensation itself, which at present is affected with much uncertainty because of the widely divergent values found by different investigators.

Kapteyn's discussion in 1908[3] of all the material then available led to the conclusion that the concentration toward the Galaxy, which is an easily demonstrated characteristic of the brighter stars, becomes more and more pronounced as increasingly fainter objects are considered. According to his figures the galactic condensation,[4] which for stars of the ninth magnitude is 2.8, rises to 5.7 at the twelfth magnitude; at the fourteenth magnitude its value is 11.5 and at the sixteenth, 27.7. Counts to the limiting magnitude of photographs of 88 Selected Areas made at Mount Wilson with the

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 135.

[2] "A Proposal for the Comparison of the Stellar Magnitude Scales of the Different Observatories Taking Part in the Astrographic Catalogue," *Monthly Notices*, **69**, 392, 1909; **72**, 464, 700, 1912; **75**, 57, 143, 465, 601, 1914–1915; **76**, 2, 149, 1915; **77**, 35, 1916. An article by Pocock in the same journal, **77**, 432, 1917, relates to the same question.

[3] *Groningen Publications*, No. 18.

[4] Ratio of number of stars per unit area at 5° galactic latitude to number at 80°.

60-inch reflector confirm substantially the relatively large value for the faint stars.[1]

The recent investigation by Chapman and Melotte,[2] on the other hand, shows nothing of the rapid increase in the ratio which is so noticeable a feature of Kapteyn's results. From the fifth to the ninth magnitude Chapman and Melotte are in agreement with Kapteyn, but at the sixteenth magnitude their value for the condensation is only 4.2, which is not greatly in excess of that for the brighter stars.

The data collected by Professor Turner contribute directly to the question of this unexplained divergence. In his earlier papers he failed to find any evidence of relatively large numbers of faint stars in the Galaxy,[3] but later the phenomenon was brought to light, and values of the coefficient of condensation were found which are of the same order as that shown by Kapteyn's tables.[4] The matter merits a closer examination, however, for, as will appear later, the Astrographic data afford a striking confirmation of Kapteyn's results as far as the limiting magnitude of the *Catalogue*, which is about 12.5 on the scale of *Groningen Publications*, No. 18.

The numerical data upon which the present discussion is based are given in the upper part of Table I as logarithms of the stellar density N_m—that is, of the total number of stars per square degree to the limiting magnitude m. The limits adopted for Table I are provisionally designated by A, B, C, and D. They are different for the different zones, and later are found to have the values given in the last line of the table. The declination of each zone appears at the head of the table; the hours of right ascension are at the left, while the galactic latitudes corresponding to the middle of each hour are alongside the logarithms of the densities. The quantities below the horizontal line will be explained later.

Comparison with the original tables in the *Monthly Notices* shows that the derivation of the quantities in Table I under the headings A, B, etc., has usually required the formation of the cumulative sums

[1] *Mt. Wilson Communications*, No. 43; *Proceedings of the National Academy of Sciences*, **3**, 217, 1917.

[2] *Memoirs of the Royal Astronomical Society*, **60**, 145, 1914.

[3] *Monthly Notices*, **75**, 607, 608, 1915.

[4] *Groningen Publications*, No. 18, p. 54, 1908.

of the numbers of stars for various intervals of brightness within each hour of right ascension. Further, it has been necessary to reduce the results to the square degree as the unit of area. For this purpose the area of an Astrographic plate has been assumed to be 4 degrees. The actual value is 169/144 times this amount,[1] but, as much work had been done before the precise value was discovered, the approximate area was used throughout. The conclusions concerning the galactic condensation are in nowise affected, and, moreover, the totals for the faintest limit in each zone, assuming them to refer to an area of exactly 1 square degree, are probably more nearly correct as they stand, because of the loss of faint stars near the corners of the plates.

The mass of material involved is considerable, the real amount being concealed under the summarized form in which it is presented; it represents the results of observations extending over several years at each of nine different observatories. The total area of the zones is nearly 6500 square degrees (that for the Vatican extends from 59° to 65°; the others are 2° wide), or approximately a sixth of the entire sky. The number of stars per zone of course varies with the declination, but ranges from about 30,000 to 80,000 or 90,000; the total is little short of 600,000, although some allowance must be made for the overlap of plates.

The following paragraphs include such details as are necessary for the derivation of the data in Table I from the tabulated counts in the *Monthly Notices*.

Vatican.—Mean counts for zones +64°, +62°, +60°; *M.N.*, **75**, 602, 1915. The original unit is one-tenth of a star per plate of 4 square degrees. Cumulative totals were formed for limiting scale-readings 30 and 0 (A and B in Table I). To reduce to densities (number of stars per square degree), 1.60 must be subtracted from the totals for each hour. Thus, for 0^h the totals are 441 and 9568; their logarithms are 2.64 and 3.98, and the corresponding densities, 1.04 and 2.38, respectively; these appear under the headings A and B in the Vatican section of Table I.

Oxford.—Zone +28°; *M.N.*, **75**, 466, 1915. The original tabular values for 0^h, 1^h, and 2^h are the numbers of stars on 7, 7, and 6 plates, with a repetition of this sequence for the remaining groups

[1] *Monthly Notices*, **72**, 466, 1912; **75**, 603, 1915.

TABLE I

LOGARITHM OF OBSERVED STELLAR DENSITY, LOG N_m

R.A.	VATICAN +62°			OXFORD +28°				BORDEAUX +17°				TOULOUSE +9°				ALGIERS −1°		
	Lat.	A	B	Lat.	A	B	C	Lat.	A	B	C	Lat.	A	B	C	Lat.	A	B
0^h.....	− 1°	1.04	2.38	−34°	0.49	1.42	1.83	−44°	0.60	1.27	1.58	−52°	0.28	0.99	1.37	−62°	0.88	1.44
1	0	1.19	2.31	−33	0.50	1.34	1.78	−44	0.48	1.30	1.56	−51	0.27	0.95	1.32	−60	0.77	1.36
2	+ 2	1.02	2.13	−28	0.64	1.33	1.78	−38	0.44	1.22	1.49	−45	0.41	0.96	1.40	−53	0.84	1.47
3	+ 6	1.00	2.07	−21	0.67	1.39	1.85	−29	0.55	1.29	1.62	−36	0.33	0.96	1.48	−42	0.92	1.55
4	+10	0.89	1.95	−12	0.37	1.14	1.59	−19	0.47	1.13	1.42	−24	0.58	1.16	1.61	−30	1.14	1.67
5	+16	0.78	2.04	− 1	0.66	1.55	2.12	− 7	0.96	1.75	2.07	−12	0.81	1.42	1.88	−16	1.31	1.80
6	+22	0.74	2.02	+10	0.65	1.80	2.39	+ 6	1.06	1.83	2.20	+ 2	1.34	1.69	2.10	− 3	1.61	2.28
7	+29	0.68	1.92	+22	0.36	1.45	2.15	+18	1.19	1.76	2.08	+15	1.08	1.49	1.96	+10	1.67	2.33
8	+36	0.56	1.69	+35	0.64	1.62	1.94	+32	1.00	1.55	1.81	+28	0.60	1.19	1.70	+24	1.44	2.03
9	+43	0.52	1.59	+48	0.49	1.65	2.02	+44	0.98	1.48	1.69	+42	0.48	1.06	1.58	+36	1.00	1.59
10	+49	0.55	1.64	+61	0.44	1.39	1.78	+58	0.85	1.35	1.52	+54	0.55	0.96	1.35	+48	1.05	1.53
11	+53	0.52	1.53	+75	0.58	1.44	1.76	+71	0.78	1.31	1.59	+64	0.53	0.94	1.35	+56	1.13	1.60
12	+55	0.34	1.54	+87	0.49	1.48	1.87	+79	0.69	1.23	1.62	+70	0.24	0.75	1.22	+60	1.01	1.47
13	+54	0.38	1.54	+79	0.52	1.45	1.75	+75	0.59	1.27	1.58	+68	0.44	0.88	1.31	+58	1.01	1.49
14	+51	0.49	1.57	+66	0.45	1.26	1.69	+62	0.78	1.26	1.50	+58	0.50	0.92	1.30	+51	0.98	1.55
15	+46	0.45	1.63	+53	0.40	1.35	1.72	+50	0.69	1.30	1.70	+46	0.55	1.03	1.52	+40	0.99	1.52
16	+39	0.54	1.72	+40	0.47	1.42	1.82	+36	0.74	1.33	1.73	+34	0.88	1.25	1.63	+28	1.15	1.66
17	+32	0.55	1.69	+28	0.70	1.58	1.92	+24	1.15	1.71	1.97	+20	0.89	1.31	1.79	+16	1.02	1.55
18	+25	0.71	1.87	+15	1.16	1.98	2.41	+10	1.07	1.74	2.05	+ 6	1.30	1.68	2.08	+ 2	1.08	1.51
19	+18	0.68	2.08	+ 3	0.88	1.90	2.45	− 2	1.16	1.84	2.17	− 6	1.12	1.52	1.99	−12	1.24	1.85
20	+12	1.00	2.07	− 8	0.85	1.86	2.31	−14	1.03	1.74	2.09	−19	1.22	1.59	1.95	−24	1.40	1.99
21	+ 7	1.06	2.13	−18	0.58	1.68	2.24	−26	0.73	1.56	1.94	−31	0.75	1.20	1.66	−37	1.26	1.76
22	+ 4	0.96	2.07	−26	0.51	1.60	2.05	−35	0.49	1.41	1.77	−42	0.82	1.20	1.62	−48	0.98	1.48
23	+ 1	1.06	2.22	−32	0.39	1.34	1.82	−42	0.47	1.35	1.71	−49	0.66	1.11	1.46	−58	0.91	1.37
.....	0	1.01	2.15	0	0.79	1.81	2.35	0	1.09	1.82	2.18	0	1.27	1.64	2.10	0	1.39	2.02
.....	10	0.94	2.10	10	0.75	1.76	2.26	10	1.05	1.77	2.12	10	1.12	1.53	2.00	10	1.37	1.97
.....	20	0.76	2.00	20	0.66	1.65	2.12	20	0.92	1.63	1.97	20	0.89	1.36	1.82	20	1.31	1.87
.....	30	0.62	1.82	30	0.59	1.51	1.97	30	0.77	1.46	1.78	30	0.72	1.19	1.67	30	1.18	1.74
.....	40	0.53	1.67	40	0.54	1.45	1.87	40	0.67	1.33	1.63	40	0.59	1.08	1.54	40	1.05	1.59
.....	50	0.49	1.58	50	0.49	1.42	1.82	50	0.62	1.27	1.56	50	0.53	1.00	1.42	50	0.96	1.49
.....	60	0.47	1.52	60	0.46	1.38	1.78	.....	.....	.....	.....	60	0.49	0.94	1.32	60	0.95	1.46
.....	.....	.....	.....	70	0.45	1.37	1.76	.....	.....	.....	.....	70	0.46	0.89	1.27	.....	.....	.....
Lim. mag.....		9.4	12.0	.....	9.2	11.4	12.5	.....	9.7	11.2	12.0	.....	9.6	10.6	11.6	.....	10.5	11.7

TABLE I—*Continued*

R.A.	Hyderabad −17°				Perth −32°				Cape −41°					Cape −42°			Melbourne −65°		
	Lat.	A	B	C	Lat.	A	B	C	Lat.	A	B	C	D	Lat.	A	B	Lat.	A	B
0h	−77°	0.37	1.36	1.69	−85°	0.61	1.16	1.51	−76°	0.76	1.08	1.48	1.59	−75°	1.21	1.52	−52°	0.81	1.48
1	−75	0.18	1.22	1.61	−78	0.53	1.11	1.45	−73	0.72	1.02	1.41	1.59	−72	1.36	1.59	−51	0.83	1.54
2	−64	0.12	1.26	1.65	−66	0.43	1.10	1.52	−64	0.72	1.02	1.36	1.62	−63	1.27	1.65	−48	0.78	1.42
3	−51	0.17	1.26	1.62	−54	0.52	1.16	1.48	−53	0.83	1.03	1.35	1.61	−53	1.38	1.70	−44	0.89	1.51
4	−38	0.37	1.32	1.76	−40	0.45	1.13	1.58	−42	0.71	1.11	1.50	1.71	−42	1.56	1.85	−38	1.06	1.67
5	−24	0.79	1.66	1.91	−28	0.62	1.26	1.73	−31	0.94	1.47	1.85	2.06	−31	1.75	2.06	−32	1.08	1.76
6	−11	0.77	1.71	2.06	−16	0.86	1.50	1.94	−20	1.07	1.45	1.89	2.05	−20	1.81	2.18	−26	1.20	1.94
7	+ 1	0.61	1.89	2.43	− 4	1.07	1.76	2.16	− 9	1.30	1.78	2.15	2.37	−10	1.94	2.30	−20	1.15	2.05
8	+14	0.49	1.59	2.12	+ 6	1.12	1.88	2.37	0	1.50	1.84	2.24	2.30	0	2.06	2.39	−15	1.22	2.02
9	+25	0.30	1.34	1.84	+15	0.99	1.70	2.12	+ 8	1.34	1.70	2.14	2.33	+ 7	1.88	2.22	−10	1.34	2.17
10	+35	0.11	1.14	1.69	+22	0.94	1.56	1.97	+15	1.00	1.52	2.01	2.23	+14	1.89	2.27	− 6	1.46	2.35
11	+42	0.11	1.16	1.77	+28	0.85	1.52	1.89	+20	0.93	1.43	1.83	2.09	+19	1.55	2.07	− 4	0.93	2.09
12	+45	0.24	1.19	1.70	+30	0.75	1.42	1.81	+22	1.00	1.40	1.88	2.04	+21	1.43	1.90	− 2	1.54	2.40
13	+44	0.40	1.30	1.85	+29	0.76	1.46	1.88	+21	0.93	1.45	1.86	2.13	+20	1.60	2.06	− 3	1.43	2.33
14	+39	0.43	1.47	2.00	+25	0.84	1.58	2.04	+17	0.91	1.39	1.89	2.21	+16	1.60	2.12	− 5	1.48	2.08
15	+31	0.43	1.46	2.03	+18	0.87	1.56	2.01	+11	1.00	1.49	1.98	2.29	+10	1.76	2.30	− 8	1.47	2.41
16	+20	0.48	1.31	1.86	+10	1.02	1.64	2.19	+ 4	1.12	1.51	1.92	2.26	+ 3	1.86	2.33	−12	1.41	2.28
17	+ 8	0.66	1.53	2.17	0	1.32	2.00	2.49	− 6	1.13	1.80	2.25	2.53	− 6	2.07	2.61	−18	0.98	1.93
18	− 4	0.44	1.48	2.19	−12	1.10	1.83	2.36	−16	1.07	1.61	2.20	2.20	−16	2.18	2.71	−24	1.23	2.17
19	−17	0.36	1.42	2.12	−24	0.87	1.48	2.00	−26	0.82	1.31	1.93	1.98	−26	1.73	2.25	−30	0.90	1.72
20	−30	0.35	1.48	2.01	−36	0.63	1.33	1.86	−37	0.79	1.22	1.75	1.84	−37	1.54	1.98	−36	0.84	1.74
21	−43	0.43	1.46	1.82	−48	0.47	1.02	1.55	−49	0.52	1.06	1.43	1.72	−49	1.42	1.87	−42	0.88	1.74
22	−57	0.34	1.31	1.74	−62	0.33	1.00	1.42	−60	0.76	1.12	1.51	1.77	−60	1.37	1.64	−47	0.86	1.65
23	−69	0.21	1.30	1.67	−74	0.69	1.35	1.62	−70	0.84	1.17	1.45	1.66	−70	1.20	1.56	−50	0.60	1.31
.....	0	0.60	1.69	2.24	0	1.10	1.82	2.32	0	1.22	1.69	2.19	2.42	0	1.97	2.39	0	1.45	2.28
.....	10	0.57	1.61	2.16	10	1.04	1.74	2.22	10	1.12	1.59	2.10	2.33	10	1.87	2.32	10	1.35	2.20
.....	20	0.48	1.52	2.01	20	0.90	1.57	2.01	20	0.98	1.45	1.93	2.15	20	1.74	2.15	20	1.17	2.04
.....	30	0.39	1.44	1.89	30	0.74	1.38	1.83	30	0.87	1.30	1.77	1.97	30	1.64	2.02	30	1.02	1.82
.....	40	0.32	1.38	1.80	40	0.60	1.24	1.69	40	0.81	1.17	1.62	1.82	40	1.53	1.90	40	0.89	1.64
.....	50	0.27	1.32	1.73	50	0.55	1.15	1.58	50	0.77	1.10	1.49	1.72	50	1.42	1.77	50	0.82	1.52
.....	60	0.24	1.27	1.67	60	0.54	1.11	1.53	60	0.74	1.06	1.44	1.65	60	1.32	1.67	60	0.77	1.46
.....	70	0.21	1.24	1.62	70	0.54	1.10	1.51	70	0.73	1.04	1.43	1.62	70	1.27	1.59	.....	.....	.....
.....	80	0.20	1.23	1.60	80	0.54	1.10	1.50	80	0.72	1.02	1.42	1.60	80	1.26	1.55	.....	.....	.....
Lim. mag.	.....	8.8	11.2	12.2	.....	9.6	11.0	12.1	.....	9.9	10.8	11.9	12.4	.....	11.5	12.4	.....	10.2	12.0

of three hours. Cumulative totals were formed for limiting scale-readings 30, 12, and 3 (A, B, and C in Table I), the last also including the values marked "Not measured." The logarithms of the totals were reduced to uniformity by adding log (7/6) = 0.07 to the results for 2^h, 5^h, etc. The constant to be subtracted to reduce the densities is log (4×7) = 1.45. The results for the other Oxford zones included in the discussion in *M.N.*, **75,** 468, cannot be used here, inasmuch as the counts have not been given.

Bordeaux.—Zone +17°; *M.N.*, **72,** 465, 1912. The original numbers are for each half-magnitude interval of the Bordeaux scale. The total number of plates is 180, an average of 7°.5 per hour; the actual number is probably 7 and 8 per hour, alternately. There is no statement as to whether the results for the various hours have been reduced to the mean area of 7°.5, but, since the correction to the logarithms is only 0.03, alternately positive and negative, the uncertainty is of no consequence and the data can be used as they stand. Cumulative totals were formed for the limits 8.5, 10.0, and 12.0 (A, B, and C in Table I). The constant to be subtracted from the logarithms to reduce to densities is log (4×7.5) = 1.48.

Toulouse.—Zone +9°; *M.N.*, **76,** 150, 1915. The original values are cumulative totals. The limits here adopted are 9.5, 11.0, and 12.3 of the Toulouse scale (A, B, and C in Table I). The totals for 13.3 are not used because of incompleteness. The total number of photographs is 180; the remarks concerning the distribution of the Bordeaux plates also apply here, and the constant is again 1.48.

Algiers.—Zone −1°; *M.N.*, **72,** 700, 1912. In the original tabulation the declination is erroneously given as +1° (*M.N.*, **76,** 155). Cumulative totals were formed for the limits 10.0 and 12.0, inclusive, of the Algiers scale (A and B in Table I). The total number of plates is 180, in groups of 7 and 8, alternately, for the successive hours of right ascension. From the remark under (*b*) *M.N.*, **72,** 702, 1912, it is inferred that the tabulated counts on p. 700 have not been referred to the mean area of 7.5 square degrees per hour. To reduce to densities, 1.45 and 1.51 were therefore subtracted alternately from the logarithms of the cumulative totals for the successive hours.

Hyderabad.—Zone $-17°$; *M.N.*, **77,** 434, 1917. The original tabulation by Pocock gives cumulative totals. Those for N_{50}, N_{20}, and N_8 (A, B, and C in Table I) were adopted. The number of plates is alternately 8 and 7 for the successive hours; but the published totals are for a constant area of 40 square degrees. To reduce to densities, 1.60 has been subtracted from the logarithms.

Perth.—Zone $-32°$; *M.N.*, **75,** 144, 1915. Cumulative totals have been formed for the limits D, G, and M, inclusive, of the Perth scale (A, B, and C in Table I). The number of plates is alternately 6 and 7, but the tabular values refer to an area of 40 square degrees and the constant is 1.60.

Cape.—Zone $-41°$; *M.N.*, **75,** 59, 1914. Cumulative totals have been formed for the limits 149, 79, -3, and -5, inclusive, of the Cape scale (A, B, C, and D in Table I). The number of plates is 6 per hour, with a total area of 24 square degrees. To reduce to densities, 1.38 has been subtracted from the logarithms of the totals.

Cape.—Zone $-42°$; *M.N.*, **76,** 3, 1915. Cumulative totals for "Measured" and "All" stars are given in the original tabulation. Their logarithms, minus the constant 1.38, appear in Table I under the headings A and B. The remaining Cape zones discussed in *M.N.*, **76,** 151, cannot be used for the present purpose.

Melbourne.—Zone $-65°$; *M.N.*, **77,** 39, 1916. The original tabulation gives cumulative totals. Those adopted are for the limits 15 and "All" (A and B in Table I). The number of plates is 3 per hour, except for hours 1, 4, 7, etc., where the number is 4. The constants for reduction to densities are, respectively, 1.08 and 1.20.

The reliability of the data collected in Table I depends mainly upon two things: first, the constancy of the limiting magnitude for the photographs of any given zone; second, the consistency of the scale of luminosity in which the results for the brightness are expressed. Thus given designations of brightness—for example, scale-readings 30 and 0 for the Vatican zone, which are the limiting values used here—must represent the same degree of brightness wherever they occur in that zone; otherwise the only method which can now be applied to these data for the investigation of the distribution of the stars with respect to the Galaxy will not give

dependable results. We have not at present the information necessary for the reduction of the brightness, referred usually to a more or less arbitrary scale, to an absolute scale of magnitudes. Consequently it is vital that the adopted limits should always mean the same thing within any given zone, and that the enumeration of the stars to that limit should be complete.

It is too much to suppose that the conditions thus implied have been rigorously satisfied. Constancy of exposure-time for all the photographs of a zone is of course presupposed, although occasional variations are known to have occurred. But, even with equality in the exposures, absolute uniformity in the limiting magnitude is not to be expected; more or less systematic fluctuations with the seasons are almost certain to occur. The sky is more transparent at one time of year than another, and, even when transparent, the atmosphere, during the winter months, is less tranquil, with a resulting diffuseness of definition in the photographic images that leads to errors of measurement. Seasonal differences in temperature and humidity modify both the sensitiveness and the gradation of the photographic plate, which in turn affect both the limiting magnitude and the consistency of the scale.

It is impossible to specify in advance the extent to which these and other disturbing factors have influenced the data with which we are concerned. Two or three circumstances affecting the completeness of the counts may be mentioned, however: As a rule, photographs for the Oxford zones were rejected unless the number of stars showing two images was at least three times the number recorded by Argelander, but this practice was not followed in the Milky Way. Only three of the plates for the zone at $+28°$ fall below this limit, however.[1] Again, the scale of brightness for the Melbourne zone exhibits a peculiarity caused by the use of different machines for the measurement of the images.[2] A considerable inconsistency affects the results, but apparently only those for the brighter stars. The difficulty has largely been avoided by choosing scale-reading 15, which is beyond the point where the large irregularity begins, as the first limit for the data in Table I. Finally, in

[1] *Astrographic Catalogue*, Oxford Section, Vol. 4, p. ix, 1908.

[2] *Monthly Notices*, 77, 36, 1916.

the case of the Hyderabad zone, there were large differences in the exposure-times and considerable variations in the sensitiveness of the plates;[1] but apparently an attempt was made to adjust these two factors in such a way as to approximate a constant limiting magnitude. The results at least seem to be as consistent as those of some of the other zones.

We shall later find evidence of irregularities in all of the zones, but for the present we assume that these exercise upon each other a compensation which reduces to a minimum their influence upon the calculated mean distribution of the stars with respect to the Galaxy.

For the derivation of the mean distribution from the values in Table I we first eliminate as completely as possible the systematic irregularities, whether observational or real in origin, as well as the accidental errors that may have affected the counts. This is accomplished by plotting the numbers in each column with their corresponding galactic latitudes as abscissae. From the smooth curves thus obtained are derived the ordinates for equidistant intervals of galactic latitude which appear below the horizontal line in the lower portion of Table I.

The dispersion of the plotted points with respect to the curves, especially when their sequence in right ascension is borne in mind, reveals at once the more conspicuous irregularities. Thus the Vatican results are closely accordant; we find nearly the same value of the density corresponding to a given galactic latitude, irrespective of the right ascension in which the region is located. The same is true of the Cape zone at $-41°$, and to a less degree of the C group for Toulouse. On the other hand, the A group in the Toulouse zone and the results for Oxford, Bordeaux, and Algiers show large variations of density with right ascension, which obviously are not directly a function of the galactic latitude. The densities for Melbourne are relatively free from systematic irregularity, but their accidental deviations seem rather large.

Later, when it has become possible to refer the data to an absolute scale of magnitudes, we shall be able to distinguish in advance many of the irregularities that are observational in origin from those

[1] *Ibid.*, 77, 433, 1917.

which represent real variations in the stellar distribution; and we shall then be in a better position to derive results for the mean distribution. At present we can only assume that the counts for regions in right ascensions differing by twelve hours have been systematically influenced in opposite directions by equal amounts. This at least is a plausible supposition so far as any seasonal effect is

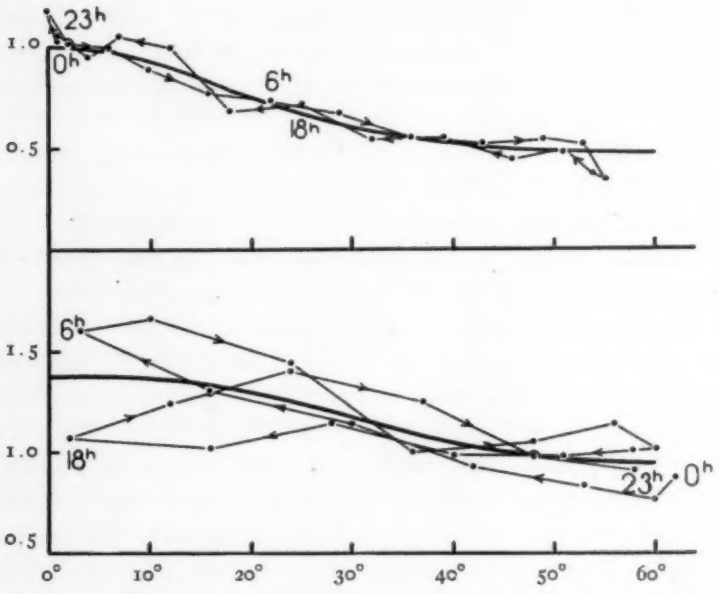


FIG. 1.—Stellar density and galactic latitude. Stars of Group A, Vatican zone, +62° (upper curve), and of Group A, Algiers zone, −1°. Note in lower curve the systematic deviations depending on right ascension.

concerned. It is on this basis that the curves have been drawn for those zones which show large variations with right ascension.

By way of illustration, two of the curves, those for the A groups of the Vatican and Algiers zones, are reproduced in Fig. 1. The broken line connects the points for adjacent hours of right ascension. For the Vatican curve there are no conspicuous irregularities, but for the Algiers results a different state of affairs exists. Note, for example, the systematic deviation in the plotted densities for 5, 6, and 7 hours from those for 17, 18, and 19 hours; yet the adopted

curve probably represents, for the zone, the mean variation of density with latitude with a tolerable degree of precision. The mean densities in the lower part of Table I should therefore be comparatively free from systematic irregularities which depend upon the right ascension.

The material has now to be combined into a final result. It may be remarked parenthetically that the mean data for each zone give directly a clear indication of the relatively high galactic condensation for the faint stars. In every case but one the difference between the values of log N_m standing in adjacent columns opposite 0° is larger than it is for the higher galactic latitudes in the last lines of Table I; in other words, with decreasing brightness the densities increase faster in the Galaxy than they do near the poles of the Milky Way.

The limits of brightness designated by A, B, C, and D were arbitrarily chosen, and thus far their precise relative values have not been determined; but for a combination of the data they must be referred to a numerical scale. We may use either that of Chapman and Melotte or that adopted by Kapteyn in *Groningen Publications*, No. 18. The latter has been chosen, since one of the purposes of the discussion is a detailed comparison of the results from the Astrographic material with the distribution tables of Kapteyn.

The transformation of the limits A, B, etc., into magnitudes depends upon the assumption that, for a given galactic latitude, equal mean densities correspond to the same limiting magnitude, whatever the position of the regions involved. Experience has proved that this supposition is reliable to a high degree, provided the areas compared are not too small; in the present case they are more than ample. With the mean densities (more precisely, the means of the logarithms of the densities) in the lower part of Table I as argument, we have only to interpolate from Kapteyn's tables,[1] for the proper latitude, the corresponding value of the magnitude. The results of this operation appear in the last line of Table I, which contains the mean of the limiting magnitudes thus derived from each of the densities standing immediately above.

[1] *Groningen Publications*, No. 18, p. 54, 1908.

In case the observed changes in density with latitude for any limit of brightness—for example, limit A of the Vatican data—were the same as that indicated by Kapteyn's tables, the result of the interpolation would naturally be the same for all the latitudes. The agreement of the individual values of the interpolated magnitudes is shown by Table II. This gives the deviation (individual *minus* mean), in tenths of a magnitude, from the mean limits at

TABLE II

DEVIATIONS FROM MEAN LIMITING MAGNITUDE
(Unit = 0.1 mag.)

Gal. Lat.	+62°		+28°			+17°			+9°			−1°	
	9.4	12.0	9.2	11.4	12.5	9.7	11.2	12.0	9.6	10.6	11.6	10.5	11.7
0°	0	−2	−2	−3	−3	−1	−1	−2	+4	+1	+1	−3	−2
10	+1	−1	−1	−2	−3	0	0	0	+3	+1	+1	−1	−1
20	0	+1	0	0	−1	+1	+1	+1	+1	+1	+1	+1	+2
30	0	+1	+1	0	0	0	+1	0	0	0	+1	+1	+2
40	−1	0	+1	+1	0	−1	0	−1	−1	0	+1	0	+1
50	−1	−1	+1	+1	+1	−1	0	−1	−2	−1	−1	−1	0
60	−1	−1	+1	+1	+1	....	....	....	−2	−2	−2	0	+1
70	....	....	+1	+2	+2	....	....	....	−3	−2	−2	....	....

Gal. Lat.	−17°			−32°			−41°				−42°		−65°	
	8.8	11.2	12.2	9.6	11.0	12.1	9.9	10.8	11.9	12.4	11.5	12.4	10.2	12.0
0°	−2	−4	−2	0	+1	0	0	0	0	−1	−1	−1	+1	0
10	−1	−3	−2	+1	+1	+1	0	0	0	0	−1	0	+1	+1
20	0	−1	0	+1	+2	+1	0	+1	+1	+1	+1	+1	+1	+2
30	+1	0	+1	0	+1	0	0	+1	+1	+1	+2	+2	+1	+1
40	0	+1	+1	−1	0	0	+1	0	0	0	+2	+2	−1	−1
50	0	+1	+1	−2	−1	−2	0	−1	−2	−1	0	0	−1	−2
60	0	+1	+1	−1	−1	−1	0	−1	−2	−1	−1	−1	−2	−2
70	−1	+1	+1	−1	−1	−1	+1	−1	−1	−1	−1	−1	....	....
80	−1	+1	+2	−1	0	0	+1	−1	−1	0	−1	−2	....	....

the head of the table which have now replaced the provisional designations A, B, etc. Though generally systematic, the residuals are remarkably small and show at a glance that the galactic condensation from the Astrographic data is substantially that found by Kapteyn; the excess of negative signs for 0° denotes a slightly brighter limit for this zone than the average, and consequently a small excess of density in the Galaxy for Kapteyn's results as compared with those from the Astrographic counts.

We are not concerned with these differences for the moment, however; the further combination of the results depends only on the mean limiting magnitude of each zone as a whole. Since these quantities are now referred to the same scale of brightness, the densities to which they correspond are directly comparable and may be united for a further elimination of the errors. Here again a graphical method is convenient. The entire series of mean densities in the lower part of Table I is plotted with the corresponding limiting magnitudes as abscissae. The smooth curve for the points of any given latitude represents the finally adopted variation of density with magnitude for that latitude. The ordinates of these curves have been collected in Table III, which for a limited

TABLE III

LOG N_m, NUMBER OF STARS PER SQUARE DEGREE BRIGHTER THAN m

Mag.	0°	10°	20°	30°	40°	50°	60°	70°	80°
8.5	0.50	0.45	0.32	0.24	0.18	0.14	0.12	0.11	0.10
9.0	0.75	0.70	0.56	0.46	0.39	0.34	0.31	0.31	0.30
9.5	1.03	0.96	0.82	0.70	0.61	0.55	0.52	0.51	0.50
10.0	1.29	1.20	1.06	0.94	0.83	0.77	0.73	0.72	0.72
10.5	1.52	1.44	1.30	1.16	1.04	0.98	0.93	0.92	0.92
11.0	1.75	1.67	1.53	1.38	1.25	1.19	1.14	1.12	1.11
11.5	1.99	1.90	1.75	1.60	1.47	1.40	1.35	1.32	1.29
12.0	2.22	2.13	1.97	1.82	1.68	1.60	1.55	1.51	1.46
12.5	2.45	2.36	2.19	2.03	1.90	1.81	1.74	1.69	1.63

range in magnitude is the equivalent of Kapteyn's table of densities. The differences between the two tables, in units of the second place of decimals, are given in Table IV. The slightly smaller densities for latitude 0° resulting from the Astrographic material are indicated by the persistent negative sign in the second column of Table IV.

A reference to the first series of curves, whose ordinates are summarized in the lower part of Table I, shows that most, if not all, of the systematic differences in Table IV are within the uncertainty of the reduction, and that the curves could be redrawn, without doing any violence to the original data, in such a manner as to remove almost entirely the systematic effect. We may therefore regard the results from the *Astrographic Catalogue* as affording

a practically complete confirmation of Kapteyn's values of the galactic condensation for the interval of four magnitudes between 8.5 and 12.5.

TABLE IV

COMPARISON WITH THE DENSITY TABLE OF KAPTEYN
(Table III *minus* Kapteyn, Unit 0.01)

Mag.	0°	10°	20°	30°	40°	50°	60°	70°	80°
8.5	−3	0	0	+2	+2	+1	+1	+1	0
9.0	−3	0	0	+1	0	−2	−2	−1	−2
9.5	−1	+1	+3	+2	0	−3	−3	−3	−1
10.0	0	+1	+3	+3	0	−2	−3	−2	−1
10.5	−1	+1	+4	+3	0	−2	−4	−2	+1
11.0	−3	0	+4	+3	−1	−2	−3	−2	−1
11.5	−3	−1	+4	+4	+1	−1	−1	0	−1
12.0	−4	−1	+3	+4	+1	0	0	+1	−2
12.5	−5	−1	+4	+5	+3	+2	+2	+1	−1

Since the value of the condensation is often expressed as the ratio of the number of stars at latitude 5° to the number at 80°, the results are also given in this form in Table V, alongside the corresponding ratios for Kapteyn and for Chapman and Melotte.

TABLE V

GALACTIC CONDENSATION. RATIO OF NUMBER OF STARS AT 5° TO NUMBER AT 80°

	Magnitude								
	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5
Astrographic Catalogue	2.4	2.7	3.2	3.5	3.8	4.2	4.7	5.4	6.2
Kapteyn	2.6	2.8	3.1	3.4	3.8	4.3	4.9	5.7	6.7
Chapman and Melotte	2.6	2.7	2.8	3.0	3.1	3.2	3.3	3.4	3.5

It is scarcely necessary to remark that the values here found for the condensation are entirely independent of the fact that we have used Kapteyn's tables for the determination of the limiting magnitudes; the result would have been the same had the different zones been reduced to a homogeneous system by the tables of Chapman and Melotte.

It is now of interest to examine the outstanding irregularities in the original data of Table I when compared with the mean densities of Table III. For this purpose the mean densities and

latitudes were first formed for 0^h and 1^h, 2^h and 3^h, etc., for the groups of each zone. Since the values in Table I refer to the middle of the hours of right ascension, the results thus derived correspond to the exact hours—1^h, 3^h, etc. The comparison with Table III gives the residuals O.—C. shown in Table VI, in units of the second

TABLE VI

DEVIATIONS OF OBSERVED DENSITIES FROM THE MEAN DISTRIBUTION
(Unit = 0.01 in the logarithm)

R.A.	+62°		+28°			+17°			+9°			−1°	
	9.4	12.0	9.2	11.4	12.5	9.7	11.2	12.0	9.6	10.6	11.6	10.5	11.7
1^h	+15	+12	− 4	−13	−18	−14	− 3	− 8	−30	− 4	− 9	−11	− 3
3	+ 7	− 8	+ 3	−29	−31	−25	−16	−20	−29	−12	− 7	−11	+ 1
5	− 3	− 8	−30	−55	−54	−29	−28	−34	−19	− 9	− 8	− 4	− 6
7	+ 2	+ 9	−22	−15	+ 1	+10	+ 7	+ 4	+19	+ 8	+ 6	+15	+26
9	− 3	− 4	+ 9	+22	+10	+27	+15	+ 4	−16	− 2	+ 6	+ 6	+12
11	+ 4	− 2	+12	+13	+ 7	+22	+12	+ 3	− 2	− 3	− 5	+12	+ 9
13	−13	− 4	+12	+22	+19	+ 5	+ 6	+13	−21	−14	−10	+ 9	+ 4
15	− 4	− 2	+ 3	− 1	− 4	+12	+ 4	+ 3	− 6	− 3	− 2	− 2	+ 3
17	− 6	− 4	+ 5	− 1	−11	+15	+ 5	+ 3	+10	+ 4	+ 3	−19	−21
19	− 5	+ 3	+21	+ 8	+ 6	+ 2	− 2	− 7	+13	+ 3	0	−32	−36
21	+13	− 3	− 4	− 5	− 3	− 3	+ 3	+ 5	+18	+12	+ 8	+17	+19
23	+ 5	− 6	−13	−12	−12	−24	+ 1	+ 3	+12	+12	+ 7	− 2	− 4
Syst. dev.	+ 1	− 1	− 1	− 6	− 8	0	0	− 3	− 4	− 1	− 1	− 2	0
Av. dev.	7	5	12	16	15	16	8	9	16	7	6	12	12

R.A.	−17°			−32°			−41°			−42°		−65°	
	8.8	11.2	12.2	9.6	11.0	12.1	9.9	10.8	11.9	12.4	11.5	12.4	10.2	12.0
1^h	+ 5	+10	+10	+ 3	+ 3	0	+ 7	+ 1	− 2	− 5	− 3	− 8	− 3	− 8
3	−10	+ 3	0	− 8	− 1	− 9	+ 9	− 4	−16	−10	− 4	− 4	− 4	−17
5	+22	+ 3	− 6	−17	−13	−14	− 1	+ 8	− 2	− 3	+14	+ 5	+10	− 3
7	+ 7	0	− 2	− 4	− 4	−12	+ 9	+10	0	− 3	+ 6	+ 2	+ 6	+ 8
9	− 6	−16	− 7	+ 6	+12	+ 7	+22	+14	+ 6	− 4	+ 2	− 6	+ 1	0
11	−20	−22	− 7	+10	+ 8	− 1	− 8	0	− 4	− 1	− 9	− 4	−14	+ 4
13	+ 4	− 7	+ 5	+ 2	+ 6	− 2	− 2	+ 1	− 3	− 3	−23	−16	+12	+16
15	+ 9	+ 6	+18	+ 2	+ 7	+ 4	−13	− 8	− 9	+ 1	−18	− 5	+15	+ 7
17	+ 3	−28	−13	+13	+11	+12	−11	0	− 8	+ 1	− 1	+ 9	− 3	+ 5
19	−20	−31	− 6	+ 9	+10	+14	− 6	+ 4	+16	− 3	+22	+36	− 1	+ 8
21	+ 6	+ 8	+ 9	−10	− 6	− 1	−11	0	− 3	− 5	+ 3	+ 9	− 6	+ 5
23	+ 5	+ 9	+ 8	− 4	+ 6	− 4	+12	+10	− 2	+ 4	− 6	− 8	−14	−14
Syst. dev.	0	− 5	+ 1	0	+ 3	0	+ 1	+ 3	− 2	− 3	− 1	+ 1	0	+ 1
Av. dev.	10	12	8	7	7	7	9	5	6	4	9	9	7	8

place of decimals. The means, both with and without regard to sign, are at the bottom, the former being the systematic deviations

of the groups, the latter the average deviation of the mean of a pair of densities, from the adopted values in Table III.

The systematic differences are unimportant and attract attention only in those cases (mainly in the Oxford zone at $+28°$) in which two or three abnormally low values were more or less ignored in drawing the original curves. Had the corresponding residuals likewise been disregarded in forming the means for Table VI, the systematic deviations would have been small throughout. The small average deviations for the Vatican zone at $+62°$ and the Cape zone at $-41°$ recall the remark of an earlier paragraph concerning the internal consistency of the results for these zones.

Examining the individual residuals, we find numerous instances of persistence of sign, which show clearly a more or less regular variation in the observed densities with right ascension. Moreover, the similarity in the sequence of signs for the different limiting magnitudes within a given zone suggests that the variations in the observed density have the same general character for stars of all degrees of brightness.

To exhibit more clearly the nature of the irregularities, the left-hand set of curves shown in Fig. 2 has been prepared. Their ordinates are the means, for each zone, of the deviations shown in Table VI for each hour of right ascension; the abscissae are right ascensions. To avoid confusion, the results for the two Cape zones at $-41°$ and $-42°$, which are much alike, have been combined. There are occasional similarities in the curves which are suggestive —for example, the first 12 hours for the zones at $+28°$, $+17°$, $+9°$, and $-1°$; but, so far as the present data alone are concerned, it is impossible to say whether the irregularities represent real variations in the distribution of the stars or nothing more than a seasonal influence upon the limiting magnitude and gradation of the photographic plates, superposed upon accidental errors of observation.

If now we examine the relation of the results thus derived to the spiral[1] found by Turner to characterize the distribution of the stars of these same zones of the *Astrographic Catalogue*, we must consider the deviations in Table VI from a different standpoint. The curve in question, whose equation is

$$\alpha + 3.66\,\delta = 247°,$$

[1] *Monthly Notices*, **76**, 7, 152, 1915.

represents the median line of a region of "obscuration" in which the ratio of the density of the faint stars to that of the bright stars is relatively small. Within any given zone—that is, for a constant declination—Turner has found the value of this ratio to vary with

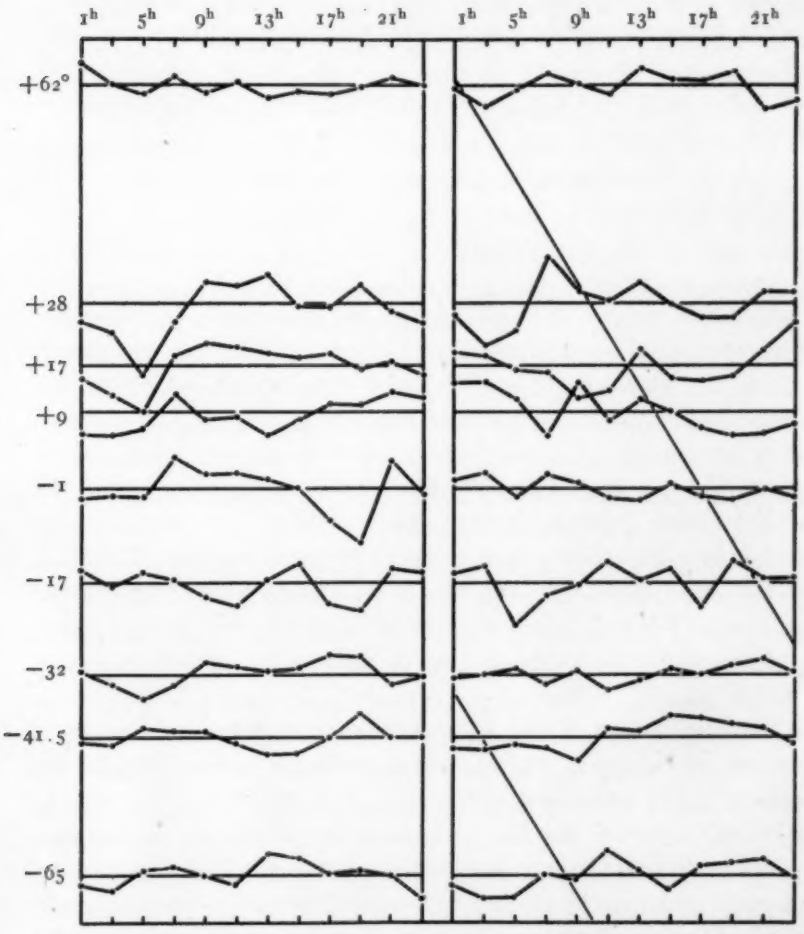


FIG. 2.—Irregularities in observed densities for various Astrographic zones. Vertical spacing of curves proportional to declination of zones given at the left; abscissae are right ascensions.

Left: Deviations from mean distribution shown in Table III. Ordinates are the means, for each hour, of the quantities in Table VI.

Right: Deviations in ratio of faint stars to bright. Ordinates are differences, last group *minus* first group, of quantities in Table VI. The median line of Turner's spiral of obscuration is indicated by the inclined lines.

right ascension between limits which depend upon the limiting magnitudes.

For a comparison with this result we may begin by forming the differences of the logarithms of the densities in Table I for the extremes of limiting magnitude there shown. It is simpler, however, to form the differences, for the extreme magnitudes in each zone, of the residuals given in Table VI. Since the deviations themselves have the form O.−C., the difference of any pair, for the limiting magnitudes F and B, may be written

$$\delta = (\log D_F - \log N_F) - (\log D_B - \log N_B) \tag{1}$$

or

$$\delta = \log \frac{D_F\, N_B}{D_B\, N_F} \tag{2}$$

in which the values of D and N are the observed and the mean tabular densities, respectively (Tables I and III), for the two limits.

It is the variation of δ with right ascension in which we are interested; D_F/D_B is the ratio of the density of the faint stars to that of the bright stars, and N_B/N_F a factor, varying with galactic latitude but not longitude, which neutralizes the influence of the relatively high galactic concentration of the faint stars toward the Galaxy. In other words, as is evident at once from Table VI, the difference in the deviations for a faint and a bright limit is the logarithm of the ratio of the number of faint to bright stars in excess of the normal ratio corresponding to the mean distribution shown in Table III.

The variation of δ with right ascension is shown by the right-hand set of curves in Fig. 2, whose ordinates were obtained by subtracting the first column for each zone in Table VI from the last, after correction for the systematic deviations at the bottom. The fluctuations are of the same order of magnitude as those shown in the other part of the figure; some similarities of form are vaguely suggested, although there is nothing that cannot be accounted for on the basis of inherent uncertainties in the data; but, whether the fluctuations are real or not, there is obviously no relation between them and Turner's spiral, which is represented by the inclined lines crossing the figure. An agreement with his results would require a well-defined minimum in each of the curves at the point of inter-

section with the spiral. Moreover, the amplitude of the variations shown in Fig. 2 is much less than that required, for his difference between the "unobscured" and "obscured" regions amounts to about 0.4 in the logarithm of the ratio of faint stars to bright.[1]

In seeking an explanation for this divergence, we may consider equation (2). Unless I have misunderstood his method of reduction, Professor Turner has treated the variations in the quantity $\log D_F/D_B$ as though they were independent of the galactic condensation; this indeed could be done, were the condensation not dependent upon brightness. Thus in *M.N.*, **76,** 3, 1915, Table I, zone $-42°$, he has given the deviations of $\log A/M$ (another notation for $\log D_F/D_B$) from its mean value; and on the following page he discusses the sequence of signs for these deviations, and for a similar series for zone $-41°$, from the standpoint of the "obscured" region. Later, these same deviations, modified by a constant, reappear in Table IV, *M.N.*, **76,** 153, 1915, where they are subjected to the analysis which leads to the spiral whose equation is given above. Again, the deviations given in *M.N.*, **75,** 468, 1915, Table III, for the differences of the logarithms of the densities corresponding to two limiting magnitudes enter unchanged into Table VI, p. 156 of Vol. **76.**[2] In neither case has there been any allowance, apparently, for the galactic phenomenon in deriving the spiral; and, similarly, in the preliminary description of the obscured patches shown in *M.N.*, **75,** 480, 1915, the discussion seems always to be based upon the unmodified ratio of the number of faint stars to the number of bright stars. In fact, only traces of the relatively high condensation of the faint stars had been detected in the Astrographic counts when the first account of the obscured areas was prepared.[3]

The quantity whose variations have been studied by Professor Turner, and upon which he has based his conclusions, is, therefore, by equation (1),

$$\delta' = \log D_F - \log D_B = \delta + \log N_F - \log N_B \qquad (3)$$

[1] *Monthly Notices*, **75,** 480, 1915.

[2] Here, as in Table IV of the same article, the first difference is for the hour of right ascension which stands at the head of the column.

[3] *Monthly Notices*, **75,** 481, 1915.

which differs from the quantity δ whose values are shown graphically in Fig. 2 by the amount $\log N_F/N_B$. But this latter quantity is not independent of the galactic latitude. Its variation is a consequence of the relatively high galactic concentration of the faint stars. Were the condensation the same for stars of all magnitudes, its value would be a constant for any pair of limits F and B; but, as we approach the Galaxy, the faint stars increase in numbers more rapidly than the bright ones, with a resulting increase in $\log N_F/N_B$.

There is a wide range in the galactic latitudes of the Astrographic zones, and the changes in the factor $\log N_F/N_B$ accounts for a large part of the variation in $\log D_F/D_B$ studied by Professor Turner. When their influence is removed, there remain only the differences δ illustrated in Fig. 2. It is thus evident that high latitudes will contribute toward the formation of an "obscured" region; a reference to the diagram in *M.N.*, **75**, 480, 1915, shows that in general the centers of these regions are remote from the Galaxy. Again, the sequence of numbers in the last column of Table IV, *M.N.*, **76**, 153, 1915, which represent the amount of the obscuration phenomenon per magnitude, is approximately duplicated by the variations in the values of $\log N_F/N_B$ interpolated from Table III for the appropriate latitudes and for limiting magnitudes differing by $1^M.0$; for example, 12.5 and 11.5. Since the curves in Fig. 2 reveal no trace of the spiral of obscuration, it seems necessary to attribute the origin of that phenomenon to the influence of the relatively high concentration of the faint stars in the galactic regions.

The question under consideration is really that of the uniformity of the stellar distribution in galactic longitude. The evidence presented here suggests that the irregularities cannot be large, at least for the fainter stars, and this conclusion is strengthened by an examination of the densities derived from the photographs of the Selected Areas made with the 60-inch reflector.[1] The limiting magnitude for these plates is about 17.5; a noticeable variation of density with right ascension is present, but it is of the character to be expected from seasonal influences. The counts on the

[1] *Mt. Wilson Communications*, No. 42; *Proceedings National Academy of Sciences*, **3**, 188, 1917.

Franklin-Adams plates,[1] limiting magnitude about 17, show even smaller systematic irregularities than those affecting the Mount Wilson photographs. Neither the Mervel Hill nor the Johannesburg series of plates reveals any variation in right ascension or galactic longitude that cannot reasonably be attributed to accidental errors.

SUMMARY

Counts collected by Professor Turner of nearly 600,000 stars on the photographs for 10 zones of the *Astrographic Catalogue*, extending from declination $+62°$ to $-65°$, have been discussed from the standpoint of stellar distribution with respect to the Galaxy. The galactic condensation found is very nearly that derived by Kapteyn (Tables III, IV, and V). The deviations of the observed densities (Table VI) from the mean distribution show systematic variations in right ascension which are definite, but not larger than the uncertainties affecting the results. They are not in agreement with the spiral of obscuration derived by Turner from the same data; the origin of this latter phenomenon is to be found in the failure to allow for the high galactic concentration for the faint stars.

MOUNT WILSON SOLAR OBSERVATORY
June 1917

[1] *Memoirs of the Astronomical Society*, **60**, 171, 1914.

THE ELIMINATION OF POLE-EFFECT FROM THE SOURCE FOR SECONDARY STANDARDS OF WAVE-LENGTH[1]

By CHARLES E. ST. JOHN and HAROLD D. BABCOCK

1. INTRODUCTION

Since the publication of the list of secondary standards of wave-length by the International Committee,[2] the attention of several spectroscopists has been devoted to the problem of adding to the number of such spectral lines. It has been sought to extend the range of wave-length covered and also to provide standards at more frequent intervals than was at first thought necessary. Determinations of wave-lengths by interference methods have been published by Burns, Meggers, and Merrill,[3] Meissner,[4] Burns and Meggers,[5] Eversheim,[6] Burns,[7] Werner,[8] and still other investigations are in progress. During the past year preliminary work in this field has been carried on at this observatory, and it is the purpose of the present paper to contribute some of the results which are now at hand.

A primary requisite which a spectral line must have in order to serve as a standard of wave-length is reproducibility. In practice this quantity depends upon two factors which determine the errors involved in locating the line in the spectrum, namely, the absolute constancy of the wave-length under the range of working conditions permitted, and the characteristics of the curve defining the intensity distribution in the line. But quite aside from the question of reproducibility, it is equally important from a practical point of view that the adopted value of a standard should be its

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 137.

[2] *Astrophysical Journal*, **32**, 215, 1910, and **39**, 93, 1914.

[3] *Scientific Papers of Bureau of Standards*, No. 274, 1916.

[4] *Annalen der Physik*, **51**, 95, 1916.

[5] *Scientific Papers of Bureau of Standards*, No. 251, 1915.

[6] *Annalen der Physik*, **45**, 454, 1914.

[7] *Journal de Physique* (5), **3**, 457, 1913. [8] *Annalen der Physik*, **44**, 289, 1914.

fundamental wave-length, that is, the wave-length free from displacements due to disturbing conditions in the arc itself that are associated primarily with proximity to the poles but are extended in range by increased current strength. It is not the purpose by this term to distinguish between a possible effect due to changes in the relative intensities of the components of a complex vibrating system and that of perturbing influences acting upon the emitting centers, or to suggest an explanation of pole effect. When, as will appear later in the paper, arc conditions are such that sensitive lines yield their fundamental wave-lengths, these lines then behave like normally stable lines. The absolute measure of pole-effect in any source is obtained by comparing the wave-lengths given by it with the corresponding fundamental values as defined above.

The introduction of the pressure-effect into the values of the secondary standards, by operating the iron arc at atmospheric pressure, is undesirable from this point of view. But while for special purposes we do, indeed, employ the arc *in vacuo* as a source of reference lines, it can hardly be recommended as a general source of secondary standards on account of its greatly diminished brightness and of the inconvenience attending its use. Moreover, the revisions of the values of the pressure-effect which are now in progress will soon supply precise data for the elimination of this factor when necessary, and the correction, being dependent in amount only upon the barometer reading, can be applied with certainty. Pole-effect, on the other hand, does not lend itself readily to such simple treatment, chiefly on account of its rapid variation from point to point in the arc and its sensitiveness to change in arc conditions. These practical considerations permit a distinction to be drawn between pole-effect and pressure-effect in regard to their bearing upon the establishment of a system of standard wave-lengths.

The phenomena of pole-effect have been studied by Goos,[1] Gale and Whitney,[2] Royds,[3] Whitney,[4] and St. John and Babcock[5]

[1] *Astrophysical Journal*, **38**, 141, 1913.

[2] *Ibid.*, **43**, 161, 1916.

[3] *Kodaikanal Bulletins*, Nos. 38 and 40, 1914.

[4] *Astrophysical Journal*, **44**, 65, 1916.

[5] *Mt. Wilson Contr.*, No. 106; *Astrophysical Journal*, **42**, 231, 1915.

under a variety of conditions, but hitherto none of the suggested explanations has met with general acceptance. While it is possible that some of the relations between pole-effect, arc-length, and current-strength, which are now quantitatively measured, may throw light upon the nature of the effect, it is not our purpose to discuss here the theories that have been proposed to account for it.

The present paper consists in an examination for pole-effect of an iron arc operated according to the specifications of the International Committee, and an experimental study of the conditions under which the error introduced by polar influence may be avoided in another form of arc.

2. APPARATUS AND METHODS

The observations discussed here were made both with the plane-grating spectrograph and with interference apparatus. The former instrument is the same as that used in a previous investigation on the pole-effect.[1] When employed in conjunction with the totally reflecting prisms and rotating sector over the slit, it has proved a powerful means of measuring minute changes in wave-length. Interference methods are also especially applicable to problems of this character; certain distinct advantages are made available by this means—namely, the possibility of obtaining a great extent of spectrum upon each photograph under strictly uniform observing conditions, the integrating action over a specified portion of a source, and the comparative insensibility to instrumental displacements. The errors inherent in this method and those which affect results derived from gratings enter in different ways, and accordance in the values reached by the two methods, used independently, is a strong indication that both are free from systematic inaccuracy.

The interference apparatus which has recently been installed in this laboratory is in general similar to that described by Fabry and Buisson.[2] The auxiliary dispersion is provided by a Rowland concave grating of 635 cm radius of curvature, which receives parallel light from a collimating mirror of nearly the same focal length, thus giving a spectrum free from astigmatism. The grat-

[1] *Loc. cit.* [2] *Journal de Physique*, **9**, 929, 1910.

ing has approximately 87,500 lines, each 50 mm long, spaced 590 to the millimeter, and provides a scale of 5 A per millimeter in the first order. Flat plates 25 cm long are used in the camera, covering 1200 A in excellent focus in the first-order spectrum, which can be observed into the infra-red as far as λ 9000. The etalon consists of fused quartz plates 40 mm in diameter, held parallel and at a fixed distance by invar separators. Silver films cathodically deposited are used for the portion of the spectrum thus far studied. The etalon is contained in a small, double-walled wooden box, and this, with the mirrors which project the rings upon the slit, is protected by an outer wooden case. The whole system stands upon the large cement pier which supports the slit and grating of the spectrograph.

Fig. 1 shows in diagram the arrangement of the parts in the optical train. The mirror M_1, of 20 cm diameter and 60 cm focal length, projects a fourfold

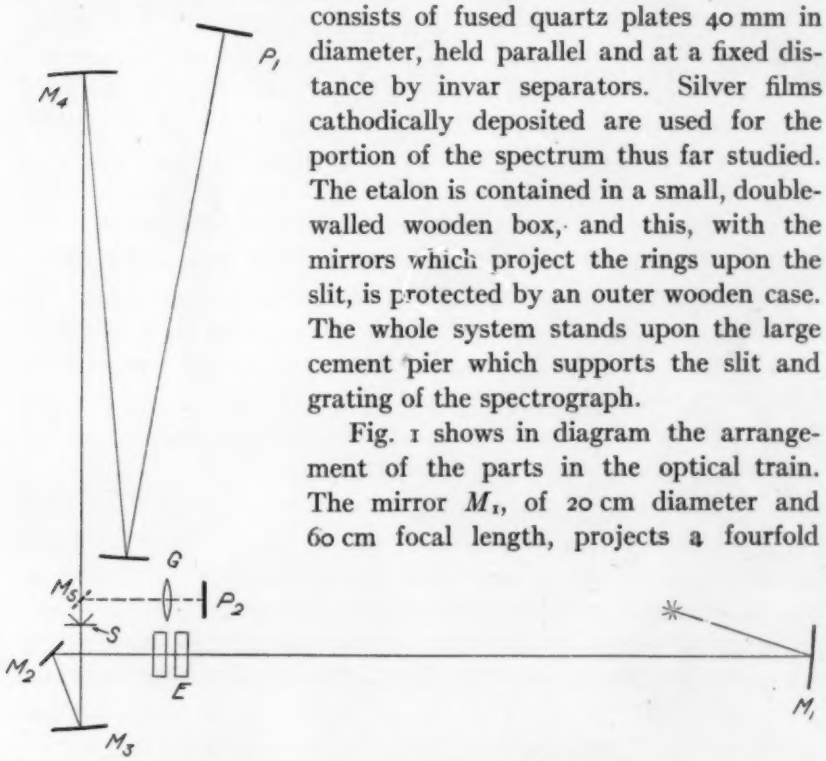


FIG. 1.—Optical system and arrangement of interference apparatus

enlarged image of the source upon the diaphragm in front of the quartz plates E. The rings are projected upon the slit S by means of the plane mirror M_2 and the concave mirror M_3. The latter has a diameter of 10 cm and a focal length of 63.3 cm. Behind the slit a plane mirror M_5 can be moved on vertical ways into or out of the beam of light which ordinarily passes to the collimator M_4, the grating G, and the plate P_1. If it is desired to photograph the red cadmium rings when the camera

is in a position which does not include that wave-length, the mirror M_5 is brought into position behind the slit so that the rings can be received in the auxiliary camera as shown. Important advantages are found in the precise achromatism of the system, in the fact that no essential part of the apparatus has to be touched between successive exposures, and in the comparative freedom from temperature disturbances. The gain in accuracy to be expected with the use of large-scale interference rings and increased auxiliary dispersion, pointed out by Pfund,[1] is realized in this apparatus, and it is doubtful whether further increase in the focal length of the projecting mirror M_3 is desirable. The focal length adopted permits the use of low magnification in the measuring microscope, with the consequent advantages in regard to the grain of the plate.

It is of interest to calculate the plate factor in angstroms per millimeter at any point in the interference system for a given set of conditions, in order to compare what one sees on photographs taken with this apparatus with the images of spectral lines formed by gratings or other instruments. It can easily be shown from the elementary theory of the interferometer that

$$\frac{pr^2}{2F^2}-E-(n-1)=0 \qquad (1)$$

where

p = order of interference at center of ring system.
F = effective focal length of lens or mirror which projects the rings on the slit, as actually affected by any magnification in the spectrograph.
E = fractional order of interference.
n = number of ring, counting innermost ring as the first.
r = radius of n^{th} ring.

Now, if wave-length λ is measured from the center of the ring system in units of spectral range, we have at the n^{th} ring

$$\lambda=n-1+E.$$

But from equation (1) we find

$$r^2=\frac{n-1+E}{4K},$$

[1] *Astrophysical Journal*, 28, 211, 1908.

where K is written for $p/8F^2$, so that we may write

$$\lambda = 4Kr^2.$$

Accordingly

$$\frac{d\lambda}{dr} = 8Kr,$$

and in order to express this quantity in angstroms per millimeter it must be multiplied by the value of the spectral range, which is λ/p. This gives

$$\frac{d\lambda}{dr} = \frac{8K\lambda r}{p} = \frac{\lambda}{F^2}r.$$

For the first ring, with the aid of equation (1), this is readily transformed into

$$\frac{d\lambda}{dr} = \frac{\lambda}{F}\sqrt{\frac{2E}{p}}.$$

If we assume

$\lambda = 5000$ A, $F = 633$ mm, $p = 40{,}000$ (10 mm etalon), $E = 0.5$,

we obtain

$$\frac{d\lambda}{dr} = 0.040 \text{ A per mm.}$$

For comparison it may be added that for λ 5000 in the fourth order of a grating having 590 lines to the millimeter, used at 9.1 meters focus, the plate factor is 0.370 A per mm, i.e., the dispersion is about one-tenth of that calculated above for the interference method.

Attention may be called to the necessity for accurate focusing of the enlarged image of the source upon the diaphragm in front of the etalon when it is desired to select any specified portion of it for study. A large relative aperture of the projecting mirror makes it possible to focus with precision, especially when the following method is used: On looking through the arc toward the projecting mirror it is easy to locate the image of the diaphragm, since the light normally incident upon the first silver film is largely returned over the path which it traversed and greatly augments the illumination. The image of the diaphragm appears, then, to float in the arc and is so much brighter than its surroundings as to be easily

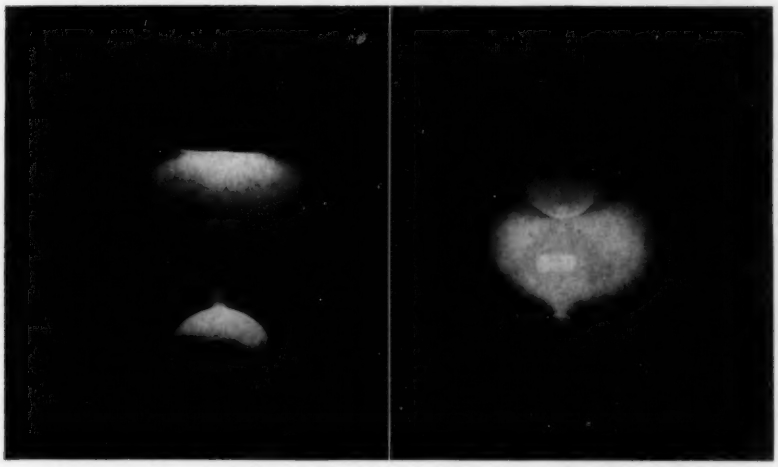
observed. It can also be seen when the arc is not burning, provided the general illumination in the room is sufficient, and its position can be very satisfactorily located by means of a hand magnifier. The electrodes of the arc, with the current cut off, can be observed simultaneously, and freedom from parallax insures a precise focus. In Plate XI are shown images of the arc, to which reference will be made later, with the diaphragm placed as in use. Owing to the fact that the arc and its image are somewhat off the axis of the projecting mirror, a small amount of astigmatism is introduced. By taking care to make the focus perfect for horizontal lines, we find that this feature becomes of advantage rather than otherwise, since it tends to smooth out the variations in intensity which exist in a horizontal section of the arc and to make the illumination of the diaphragm more uniform while leaving the extent of the arc-axis in use sharply defined.

The observations to be described were obtained for the most part with a diaphragm measuring 5×12 mm, but for special purposes others of circular shape are employed, having diameters of 5, 9, and 12 mm. The rectangular diaphragm is especially useful when it is necessary to economize light while using only a limited extent of the axis of the arc.

To test the auxiliary spectrograph for distortion of the field of view in a direction parallel to the spectral lines, a plate was placed over the slit, having a series of ten narrow openings arranged to simulate the relative positions of the sections of interference rings formed by the slit. With the etalon removed an exposure was made to the spectrum which reproduced each line broken into ten dots, with much the same appearance as though produced by interference. An examination of the distances separating these dots for lines in different portions of the negative and a comparison with the distances of the slots in the occulting plate showed constant magnification and freedom from distortion in all parts of the field.

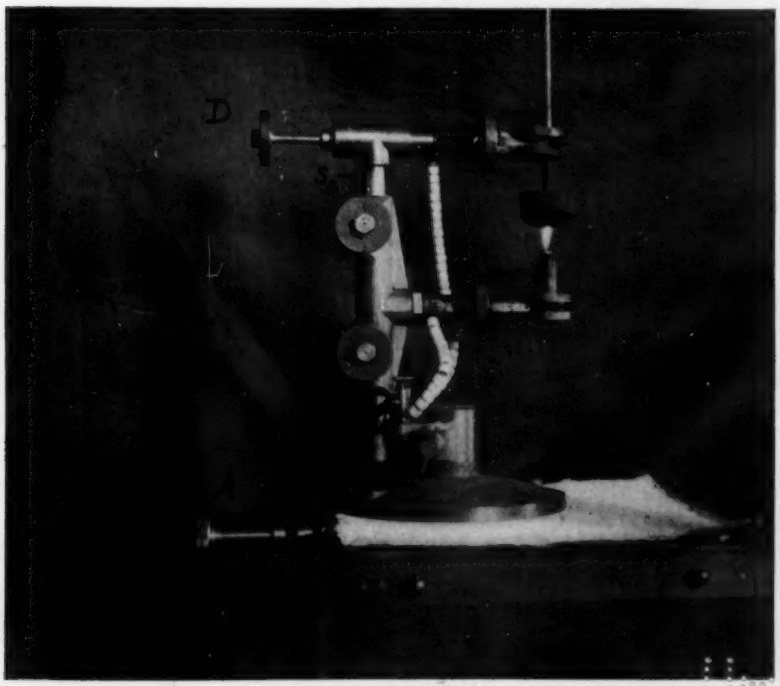
The method of observation is essentially that of Fabry and Buisson. The etalon is usually adjusted with the aid of the green mercury line by observing the rings in a telescope while a very small exploring diaphragm is moved about in the beam

PLATE XI



a *b*

DISTRIBUTION OF LUMINOSITY IN 12-MM, 5-AMP PFUND ARC (*a*) AND IN INTERNATIONAL ARC (*b*) WITH SUPERPOSED IMAGES OF DEFINING DIAPHRAGM



ARC LAMP FOR PRECISION OBSERVATIONS

of light. When the adjustment for parallelism is complete with large aperture, the diaphragm is inserted very close to the first plate without touching any sensitive part of the instrument, and the protecting cover is put in place. The exposures are so distributed as to provide a check upon the constancy of the etalon.

The use of standard distance-gauges over the slit for the purpose of reducing linear diameters to angular measure is avoided, the observed diameters themselves furnishing all the necessary data as follows: The diameters are measured upon a machine provided with two precision screws whose axes are at right angles, from two to five rings being observed, according to the nature of the problem at hand. When four or five diameters are observed, the fractional order may be most accurately found from a least-squares solution of the observational equations, which are written in the form of equation (1) above. For differential observations upon the wave-length of the same line under slightly different conditions two rings are generally observed, and a plate-constant, $K\lambda$, is determined. It is the average of all the values obtained by dividing each approximate wave-length by the difference in the squares of the diameters corresponding to it. The value of K for any line is then taken by redivision, and the fractional order is quickly obtained from each diameter.

The integral order of interference and the correction for change of phase are readily found by well-known methods, a convenient arrangement of which is described by Meggers.[1]

The arc lamp is supported upon a small wooden platform, which is provided with a coarse screw for moving it at right angles to the line of sight. This permits easy correction of position without disturbing the setting for focus. Details of the mounting are shown in the lower half of Plate XI. The guiding screw referred to is marked A. The base of the lamp is heavy enough to give sufficient stability, and the moving parts are well defined in position. Screw B provides a vertical motion of the entire upper part of the lamp, read on a millimeter scale S_1. The upper electrode is moved up and down relatively to the lower by screw C, the scale S_2 serving to show

[1] *Scientific Papers of Bureau of Standards*, No. 251, 1915.

the arc-length. For centering the electrodes screw *D* provides a separate horizontal motion for the upper terminal. Current for the arc is supplied by direct-current generators at a maximum of 300 volts. The voltages commonly used are 110 and 220, with lamp resistance in series with the arc for control. An ammeter is kept in circuit and the current is read to 1/5 ampere.

3. OBSERVATIONS

a) Provisional standard source.—In order to examine a given source for the presence and amount of pole-effect contained in it, a reference source practically free from this disturbing influence is

TABLE I

POLE-EFFECT IN PFUND ARC WITH CARBON NEGATIVE POLE
IRON LINES OF GROUP *d*

Plate	Negative Pole *minus* Center		Plate	Positive Pole *minus* Center	
	Number Lines	Mean Shift		Number Lines	Mean Shift
		A			A
1	29	+0.003	1	38	+0.007*
2	31	.000	2	38	.008*
3	29	+ .001	3	37	.002
4	32	.000	4	36	.003
5	32	− .001	5	38	.003
6	36	.000	6	37	.002
7	36	− .001	7	37	+0.003
8	36	0.000			
Mean		0.000	Mean		+0.004

* No sector used.

essential. Our former conclusion, that the center of a 6-mm, 6-ampere Pfund arc is free from pole-effect,[1] is incorrect, as will be shown below, and a substitute was sought in the Pfund arc having the negative pole of carbon. Such an arc has been tested for pole-effect and found suitable for the purpose. The length of arc was maintained between 6 and 8 mm, and the current was 6 amperes on the average. The measurements summarized in Table I were made on plates obtained with the 30-foot plane-grating spectrograph, using sector and prisms over the slit. The

[1] *Astrophysical Journal*, 27, 298, 1906.

lines observed all belong to group *d*. For these lines no difference is found between wave-length at negative pole and at center, while a comparatively small displacement appears at the positive pole. A narrow central zone of this arc was accordingly adopted provisionally as a standard source, assumed to be free from pole-effect.

b) *Pole-effect in center of international arc.*—In accordance with the specifications of the International Committee on Wave-Lengths,[1] an arc 6 mm long between iron rods 7 mm in diameter, fed by a current of 6 amperes, under a pressure of 220 volts, having

TABLE II

CENTER INTERNATIONAL ARC *minus* CENTER PFUND CARBON-POLE ARC
IRON LINES OF GROUP *d*

Plate	Number Lines	Mean Shift	Plate	Number Lines	Mean Shift
		A			A
1.....	21	+0.004	7.....	34	+0.005
2.....	19	.006	8.....	33	.006
3.....	14	.010	9.....	34	.006
4.....	33	.007	10.....	32	+0.004
5.....	23	0.008			
6.....	38	+0.006	Mean...	.....	+0.006

Mean pole-effect in 6-mm, 6-ampere Pfund arc for the same lines is +0.010 A.

its positive pole above, will be referred to in this paper as the international arc. Light is taken from a central zone not exceeding 2 mm in width.

The slit of the 30-foot spectrograph was placed at right angles to the axis of this arc at its middle point, as recommended for finest definition in the specifications of the committee, and by means of a small totally reflecting prism simultaneous exposures were made upon this source and upon the corresponding part of the provisional standard source. A rotating sector equalized the effective intensity upon all parts of the slit, and special care was taken to see that each light-source completely illuminated the grating. As shown in Table II, ten exposures give for some thirty sensitive lines in the region λ 3840–λ 4200 a mean value of +0.006 Å for the difference international *minus* Pfund carbon-pole arc. The

[1] *Trans. International Union for Co-operation in Solar Research*, 4, 59, 1914.

conclusion indicated is that the international arc has pole-effect, even in its central section, at least 0.006 A in amount.

c) *Distribution of pole-effect in international arc.*—Spectra from near the negative pole of the international arc and from its center were compared by means of the 30-foot spectrograph, using the method of simultaneous exposures already referred to. The difference in brightness of the two parts of the arc was compensated by a rotating sector; control measurements made upon stable lines

TABLE III

INTERNATIONAL ARC, NEGATIVE POLE *minus* CENTER

Plate	Normal Polarity, Pos. Pole Above				Plate	Inverted Polarity, Neg. Pole Above			
	Group *d*		Group *e*			Group *d*		Group *e*	
	Number Lines	Mean Shift	Number Lines	Mean Shift		Number Lines	Mean Shift	Number Lines	Mean Shift
		A		A			A		A
1.....	34	+0.021	3	−0.012	1.....	19	+0.018	1	−0.004
2.....	40	.019	3	.012	2.....	21	.013	2	.004
3.....	40	.018	3	.013	3.....	39	.010	3	.001
4.....	37	.014	3	.009	4.....	38	.008	2	.004
5.....	34	.018	3	.012	5.....	39	.013	3	.008
6.....	33	.019	3	.011	6.....	41	.010	3	.009
7.....	33	+0.017	3	−0.010	7.....	41	.011	3	.012
					8.....	41	.013	3	.010
Means	.....	+0.018	.....	−0.011	9.....	40	.013	3	.009
					10.....	40	.010	3	.007
					11.....	40	+0.013	3	−0.005
					Means	.....	+0.012	.....	−0.007

Pole-effect in 6-mm, 6-ampere Pfund arc for the the same lines: Group *d*, +0.011 A; Group *e*, −0.009 A.

showed freedom from instrumental displacement; the distinguishing feature of the pole-effect appeared in the opposite displacements given by lines of groups *d* and *e*. From 35 to 40 sensitive lines between λ 3800 and λ 4200 were observed upon each of seven plates, the mean values obtained being shown in the left-hand part of Table III. The polarity of the international arc was then reversed, making the upper pole negative, as it is in the Pfund arc, and the difference negative pole *minus* center was again observed in the same manner. The results of these measurements are collected in the right-hand part of Table III, which is self-explanatory. The mean pole-effect measured in the Pfund arc for the same list of lines is added at the bottom of the table for comparison. These

observations leave no doubt as to the existence of pole-effect in the international arc in somewhat greater amount than in the 6-mm, 6-ampere Pfund arc. The amount of material discussed in the case of the *d* lines is thought sufficient to give real meaning to the difference in the two mean values in Table III; the investigations bearing upon this point are still in progress.

The interference method was next employed for comparing integrated wave-lengths over the central half of the international arc with the corresponding values from the provisional standard source described above. A suitable diaphragm, placed in front of the etalon, selects the desired portion of the source, and since with proper precautions the instrument integrates all the light falling upon the diaphragm it is well suited to such a purpose. Under these conditions it was found that the central half of the international arc gave wave-lengths for 30 sensitive lines greater on the average by 0.005 A than the corresponding portion of the Pfund arc having the negative pole of carbon, although 59 stable lines showed no change in wave-length between the two sources. Furthermore, a comparison of the central half of the international arc with the central third of the same arc was made by a suitable exchange of diaphragms. In this case a list of 35 lines known to be free from pole-effect showed no variation, but 35 sensitive lines of group *d* showed a mean decrease of 0.003 A when the smaller portion of the arc was used. These facts are most readily explicable on the assumption that the central half of the international arc exhibits pole-effect, and a comparison of the two mean differences given above indicates that even the recommended central third of the international arc is somewhat affected by the polar influence. Although these data are not sufficient to afford a reliable measure of the amount of pole-effect in the central third of the arc, they tend to confirm the more exact quantitative measures presented above in section *b*.

Tests made toward the close of the investigation by the method described in the following section show that when the international arc is lengthened to 12 mm a central zone $1\frac{1}{4}$ mm wide still contains measurable pole-effect. The lines of a well-known list belonging to group *e* are, on the average, displaced

toward the violet 0.004 A from the positions which they have when free from pole-effect. The lines of group *d* which were tested show less displacement, but, on the whole, a tendency to shift toward the red.

d) *Development of the Pfund arc for standard purposes.*—The source described above as a provisional standard, the Pfund arc with carbon negative pole, is for many purposes all that could be desired. It is especially useful in the ultra-violet, where it yields most of the strong iron lines unreversed and very narrow. No difficulty is experienced from the so-called carbon bands when only the central region of the arc is used, and, as Pfund says, it yields iron lines "rivaling in homogeneity and sharpness the red and green lines of cadmium produced in a vacuum arc."[1] But its intensity, especially in the region of longer wave-lengths, is relatively feeble. The international arc, on the other hand, has been found characterized by a troublesome amount of pole-effect, which necessitates the introduction of uncertain correction terms before it can be safely employed as a reference source in many kinds of spectroscopic problems. A study has been made of the Pfund arc, accordingly, in order to develop its possibilities as a source suitable for a standard.

The interference method is so well suited to such a problem that extensive use has been made of it. For the most part a 5-mm etalon was used, this rather short length being chosen in order that the widest lines, which are in some cases most sensitive to pole-effect, might give measurable interference fringes. A diaphragm in front of the etalon selected a horizontal section through the center of the arc $1\frac{1}{4}$ mm wide.

The method usually employed for differential interference observations was abandoned, partly on account of the prolonged exposures sometimes necessary, and also because of the saving of labor and greater accuracy made possible by the plan adopted. The procedure followed consisted in determining the wave-lengths of the sensitive lines of groups *d* and *e* from each source in terms of selected stable lines occurring upon the same plate. Three sharp *a* lines in the green, which by comparison with the green mercury

[1] *Astrophysical Journal*, 27, 298, 1906.

line have been found to be constant in wave-length under varying arc conditions, were adopted as reference standards.

It should be noted that the existence of pole-effect in small or moderate amounts is not necessarily accompanied by any suggestive appearance of the spectrum lines, such as unsymmetrical widening. The only certain test for the effect is the actual measurement of the wave-length. The differences shown in the accompanying tables cannot be attributed to unsymmetrical widening dependent upon increase of intensity, or to an "intensity equation" directly, since the individual lines exhibit a much greater range of intensity upon each plate than a given line shows from one plate to another, while the differences of wave-length are substantially the same for all lines of a given group. Burns, Meggers, and Merrill conclude[1] that "interferometer wave-lengths are comparatively unaffected by any intensity equation," and, so far as the present data bear upon this question, they seem to confirm that conclusion.

While the method employed eliminates systematic instrumental errors from these measurements, the objection may be raised that the observer tends to set the cross-wire of his microscope in a systematically different way upon lines belonging to different groups. All of the photographs at present under discussion, however, resemble one another very closely; they were taken with the same etalon, had approximately equivalent exposures, and were measured by the same observer, so that if such a source of error is present at all it is constant upon this series of plates. The differences that are shown in the behavior of these sensitive lines appear to arise, therefore, from actual changes in the wave-lengths of the lines.

Reference has already been made to a former paper in which the pole-effect was studied under certain conditions of arc-length and current-strength for this type of arc. In that investigation the center of such an arc was used for the working standards, but the tests made to ascertain whether it was itself entirely free from polar influence are now seen to be inadequate. In Table IV are given the results of a comparison of the center of the 6-mm,

[1] *Scientific Papers of Bureau of Standards*, No. 274, 1916.

6-ampere Pfund arc with the same region of the Pfund arc having the negative pole of carbon. A small amount of pole-effect is clearly indicated for lines belonging to both groups *d* and *e*.

A visual examination of the Pfund arc through a dense neutral screen shows a clearly defined structure consisting of the outer layer, which owes its luminosity principally to the band spectrum of iron oxide, a darker space within, which emits very little of the red end of the spectrum, and two brilliant streams of vapor in the axis of the dark space, of which the one proceeding from the negative pole is by far the brighter. Traces of

TABLE IV

POLE-EFFECT IN CENTER OF 6-MM, 6-AMPERE PFUND ARC

GROUP *d*			GROUP *e*		
Carbon Neg. Pole	Pfund	Shift	Carbon Neg. Pole	Pfund	Shift
5232.950....	.952	+0.002	5383.377...	.371	−0.006
5266.568....	.570	.002	5404.146...	.140	.006
5324.190....	.192	.002	5410.918...	.912	.006
5569.628....	.631	.003	5415.204...	.196	.008
5572.852....	.855	.003	5424.073...	.065	−0.008
5586.766....	.767	.001			
5615.653....	.656	.003			
5624.552....	.557	+0.005			
Mean..	.....	+0.003	Mean..	.....	−0.007

this structure are shown in Plate XI*a*. This photograph of the Pfund arc was made with light from the spectral region λλ 5800–7000, and the original contrasts have been considerably reduced by means of ammonium persulphate. Plate XI*b*, on the other hand, shows the international arc photographed in the same way with no reduction of the scale of contrast. The original exposure in this case was very much less than for the Pfund arc. If such photographs are taken with blue and violet light, the outer mantle is found much reduced, and the inner "dark" space, referred to above, is very much increased in relative intensity. Our observations upon pole-effect have led us to associate it with the bright streams of vapor which issue from the poles, and it was therefore natural

to choose the comparatively dark region between the adjacent ends of these vapor streams as a source presumably free from pole-effect. In the 6-mm, 6-ampere Pfund arc, however, as was shown above, this procedure is not permissible, but the observations next to be described show that simple modifications of the arc attain the desired result.

A normal Pfund arc 12 mm long was operated at varying current-strengths under a pressure of 220 volts. It was found that with currents of from 4 amperes to 7 amperes the arc was remarkably steady, and any desired portion of it could be selected

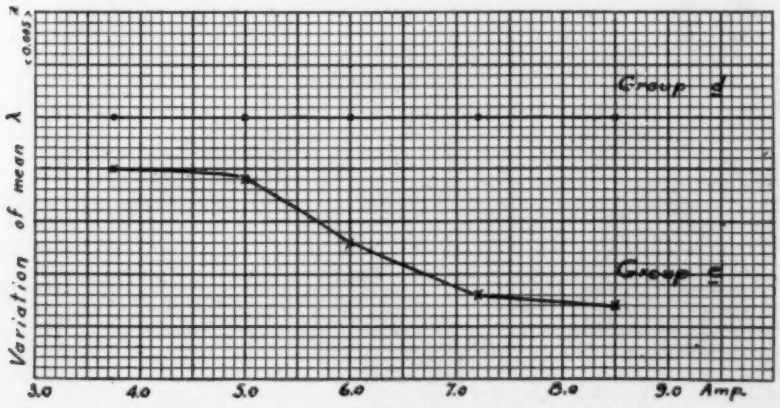


FIG. 2.—Relation between wave-length and current strength for sensitive lines

with considerable accuracy; exposures of an hour's duration could be made without restriking the arc. With further increase of current, intervals of unsteadiness appeared, but satisfactory plates could be taken up to 8.5 amperes. For each current-strength employed the wave-lengths of a series of *d* lines and of certain *e* lines were determined in terms of the same three standard *a* lines according to the method described above. The results are displayed in Table V and in the diagram, Fig. 2. The wave-lengths have had applied to them the necessary small corrections for phase-change.

A comparison of the means at the bottom of the columns shows striking uniformity for the *d* lines, but for currents above 5 amperes very definite displacements for the lines of group *e*. Special

attention should be called to the fact that, for currents of 5 amperes or less, even the sensitive *e* lines show practically constant values. The obvious interpretation of these results is that for the arc specified a central zone at least $1\frac{1}{4}$ mm wide is free from pole-effect when the current is less than 5 amperes, and wave-lengths derived from it have their fundamental values.

TABLE V

RELATION OF WAVE-LENGTH AT CENTER OF 12-MM PFUND ARC TO CURRENT-STRENGTH STANDARDS λλ5397.132, 5405.781, 5434.528

λ	CURRENT IN AMPERES					MEANS
	3.75	5.00	6.00	7.20	8.50	
			GROUP *d*			
5232.....	.950	.950	.951	.950	.952	.951
5266.....	.568	.567	.565	.567	.563	.566
5324.....	.190	.188	.189	.188	.186	.188
5569.....	.629	.628	.629	.627	.627	.628
5572.....	.852	.851	.851	.852	.851	.851
5586.....	.767	.765	.765	.767	.766	.766
5602.....	.954	.955	.953	.954	.954	.954
5615.....	.652	.653	.655	.655	.655	.654
5624.....	.552	.556	.549	.552	.553	.552
5488.....	.457	.457	.457	.457	.457	.457
			GROUP *e*			
5383.....	.379	.377	.372	.370	.371	.....
5404.....	.148	.146	.138	.135	.132	.....
5410.....	.918	.920	.912	.910	.908	.....
5415.....	.205	.205	.198	.188	.190	.....
5424.....	.075	.073	.070	.062	.059	.....
5407.....	.545	.544	.538	.533	.532	.....

Confirmation of this conclusion is afforded in Table VI, in which the second column presents for group *d* means taken from all the measures in Table V, and for group *e* the means from the two lowest currents given in that table; the third column gives the results for the same lines in the carbon-pole Pfund arc, in terms of the same standards. The succeeding columns add corresponding data from Pfund arcs having negative poles of copper, brass, and chro-

mium steel, respectively. The arc-length was nearly 12 mm, and the current was approximately 5 amperes in each case. The variations in the means are undoubtedly within the errors of measurement, as it must be remembered that the lines concerned are among the most difficult of all iron lines to measure. Since the pole-effect is found to have its greatest value in the vicinity of the negative pole, it seems improbable that such accordance would be found if the values in the second column obtained with the iron pole were affected by it, as this hypothesis would require exactly equal amounts to appear in the other arcs considered.

TABLE VI

SENSITIVE LINES INDEPENDENT OF CONSTITUTION OF NEGATIVE POLE
CENTER OF 12-MM, 5-AMPERE PFUND ARC

λ	NEGATIVE POLE					WTD. MEAN
	Iron	Carbon	Copper	Brass	Cr. Steel	
			GROUP *d*			
5232.....	.951	.950	.950	.949	.950	.950
5266.....	.566	.568	.566	.566	.565	.566
5324.....	.188	.190	.190	.190	.189	.189
5569.....	.628	.628	.628	.630	.627	.628
5572.....	.851	.852	.852	.851	.852	.851
5586.....	.766	.766	.765	.764	.768	.765
5615.....	.654	.653	.653	.655	.652	.654
5624.....	.552	.552	.554	.552	.552	.552
5474.....	.144	.145	.145	1.45	.144	.....
			GROUP *e*			
5383.....	.378	.377	.377	.378	.378	.378
5404.....	.147	.146	.145	.144	.143	.145
5410.....	.919	.918	.917	.917	.920	.918
5415.....	.205	.204	.206	.205	.206	.205
5424.....	.074	.073	.073	.074	.074	.074
5407.....	.545	.544	.544	.544	.544	.....

It is found by examining monochromatic images of the Pfund arc, formed by various spectrum lines, that the distribution of luminosity in the iron vapor is practically independent of the constitution of the negative pole, as far as such observations have been

made. Under these conditions there is a slight increase in the brightness of the iron vapor close to the pole in question, but it is not prominent. The brilliant stream issuing from this electrode, on the other hand, shows in general the same characteristic distribution for the various substances which we have used for cathodes in our mixed arcs. In each case there is great increase of intensity close to the negative pole. Since it has been shown above that, in the case of the Pfund arc having the negative pole of carbon, the pole-effect disappears in the vicinity of the cathode, we have evidence here which suggests that it is not mere proximity to the negative pole which produces the effect, but that an essential condition is that the emitting centers originate with this stream. Our experience in obtaining photographs for measurement on pole-effect amply confirms this idea, for when the prism selecting the light at the pole is given a slight lateral displacement out of this narrow stream, the observed pole-effect is greatly diminished.

Furthermore, in a former investigation, already referred to, it was found that when the Pfund arc is operated *in vacuo* the pole-effect disappears and that the brilliant vapor stream which issues from the negative pole at atmospheric pressure is lacking, the arc appearing as a soft nebulous glow, brighter, indeed, near the negative pole, but without definite structure. In the light of these observations it is difficult to see how pole-effect can be directly dependent upon anything in the nature of electronic bombardment of the metal vapor by particles from the negative pole.

The gradual appearance of pole-effect in the center of an arc as its length is decreased may be seen in Table VII. The current was kept constant at 5 amperes, and arc-lengths of 12, 10, 8, and 6 mm were employed, the method of observation being the same as that referred to in the preceding paragraphs. The lines of group *d* are shown to remain practically constant in wave-length under these conditions, but the *e* lines, which offer a more sensitive test for the presence of pole-effect, show it definitely for lengths less than 8 mm. By a combination of the data presented in Tables V and VII it is found that, in order to be free from pole-effect in the center of the Pfund arc, the length must be at least 8 mm and the current not over 5 amperes. Under these condi-

tions a horizontal zone at least $1\frac{1}{4}$ mm wide may be used with safety, even for observations upon the most sensitive lines of group *e*. For *d* lines, on the other hand, the arc-length may be as short as 6 mm for a current of 5 amperes, or the current may be as much as 8.5 amperes for a length of 12 mm without introducing an appreciable amount of pole-effect.

TABLE VII

RELATION OF POLE-EFFECT TO ARC-LENGTH FOR 5-AMPERE PFUND ARC
CENTER OF ARC USED

λ	Arc-Length in Millimeters			
	6	8	10	12
Iron Lines of Group *d*				
5232.....	.951	.950	.949	.950
5266.....	.566	.569	.566	.567
5324.....	.189	.191	.189	.188
5569.....	.626	.628	.629	.628
5572.....	.850	.852	.852	.851
5586.....	.763	.765	.765	.765
5615.....	.652	.653	.654	.653
5624.....	.552	.552	.552	.556
5474.....	.144	.145	.144	.145
Iron Lines of Group *e*				
5383.....	.372	.375	.376	.377
5404.....	.141	.144	.144	.146
5410.....	.912	.916	.916	.920
5415.....	.197	.205	.204	.205
5424.....	.063	.071	.072	.073
5407.....	.537	.542	.542	.544

A set of conditions under which the Pfund arc may be used without introducing pole-effect having been developed, it is of interest to examine the wave-lengths of lines in the adopted list of international secondary standards known to exhibit pole-effect. In Table VIII, the first column gives the value published by the International Committee, the second shows the fundamental value obtained under these conditions, and the third contains the differences. The Mount Wilson values are not final, but represent our best knowledge of these lines at present. Definitive wave-lengths

covering a wide range of spectrum will appear in a later paper. The mean difference shown in Table VIII is practically coincident with the mean difference obtained above in Table II, although the observations involved in the two cases are entirely independent.

TABLE VIII

POLE-EFFECT IN INTERNATIONAL SECONDARY STANDARDS

I.A.	Mt. Wilson	I.A. *minus* Mt. Wilson	I.A.	Mt. Wilson	I.A. *minus* Mt. Wilson
4707.288....	.282	+0.006	5232.957...	.949	+0.008
4736.786....	.782	.004	5266.569...	.566	.003
4859.758....	.752	.006	5302.315...	.309	.006
4878.225....	.219	.006	5324.196...	.188	.008
4903.325....	.318	.007	5569.633...	.627	.006
4919.007....	.001	.006	5586.772...	.765	.007
4966.104....	.096	.008	5615.661...	.653	+0.008
5001.881....	.873	.008			
5192.363....	.356	+0.007	Mean...	.....	+0.0065

In Table IX we compare our values with the determinations by Burns, Meggers, and Merrill[1] for lines of group *d*. The mean indicated makes it clear that pole-effect played a part in their

TABLE IX

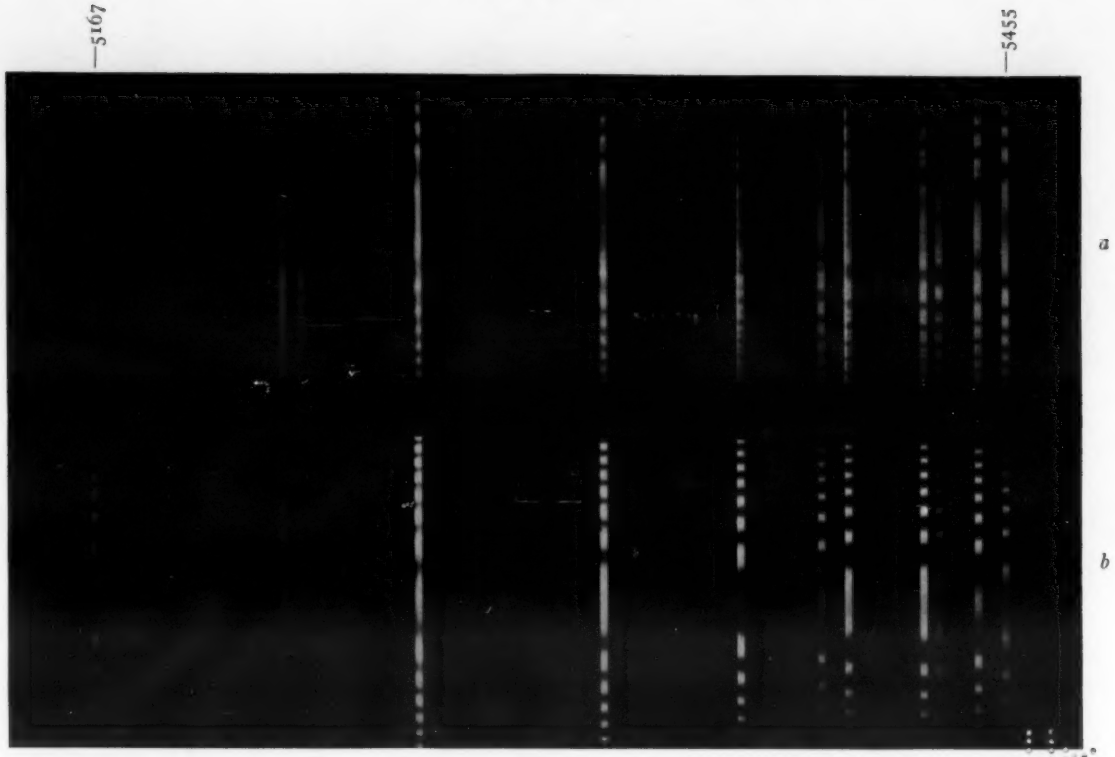
POLE-EFFECT IN RECENT INTERFEROMETER OBSERVATIONS—GROUP *d*

Mean λ free from Pole-Effect	Burns, Meggers, Merrill	Difference	Mean λ free from Pole-Effect	Burns, Meggers, Merrill	Difference
5232.949....	.957	+0.008	5586.765...	.772	+0.007
5266.566....	.572	.006	5615.653...	.661	.008
5302.309....	.315	.006	5624.552...	.559	+0.007
5324.188....	.195	.007			
5569.627....	.633	.006	Mean...	.....	+0.007
5572.851....	.859	+0.008			

investigation, and that its amount was substantially the same as that existing in the international secondary standards. This latter fact is sufficient to account for the failure of these observers to find evidence of it in their results, as they made no comparisons with a source free from this influence. They say in their paper:[2]

[1] *Scientific Papers of Bureau of Standards*, No. 274, 1916. [2] *Ibid.*

PLATE XII



COMPARATIVE SHARPNESS OF LINES FROM INTERNATIONAL ARC (*a*) AND PROPOSED ARC (*b*); 15-MM ETALONS

"Thus it seems probable that the measurement of additional lines on the plates from which the I.A. standards were obtained would lead to the same values of the wave-lengths as though the plates had been taken under normal conditions." This conclusion is confirmed by the observations presented above, but it must be sharply distinguished from the statement in the summary at the end of their paper to the effect that "interferometer observations under the conditions described were found to be reasonably free from the effects of pole shifts." No evidence was adduced by them in support of this statement, and in the light of the present data it is seen to be unwarranted.

e) Comparative sharpness of iron lines in different arcs.—An apparent superiority of the international arc over the Pfund type lies in its increased intensity for unreversed lines, but this increase of intensity is accompanied by a corresponding increase in width of the lines themselves, as quantitatively measured by Nutting,[1] so that the advantage gained in exposure time is offset by the disadvantage of lower reproducibility in the international arc. The relative sharpness of certain lines in the region λλ 5167–5455 in the international arc and in the Pfund arc when free from pole-effect is shown in Plate XII, in which *a* is taken from the center of the international arc and *b* from the Pfund arc. The etalon used was 15 mm long, i.e., the order of interference is about 55,000. The diaphragm admitted from the center of the arc an equatorial section $1\frac{1}{4}$ mm wide, while the specifications permit the use of 2 mm in the case of the international arc. If this larger amount had been used, the difference between the two arcs would have been still further increased. The prominent lines of group *a* are shown reversed in the international arc, and there is practically no material suitable for measurement upon the plate from which this illustration was made. Upon a plate from the Pfund arc free from pole-effect, however, we are able to measure all lines of sufficient intensity, and a still higher order of interference could be used. The number of lines which reverse in the international arc, but which are unreversed and suitable for measurement in the Pfund arc, is still greater in the region of shorter wave-lengths.

[1] *Bulletin of Bureau of Standards*, 2, 439, 1906.

4. THE IMPORTANCE OF FUNDAMENTAL WAVE-LENGTHS

The main purpose of the new determinations is to obtain extensive lists of wave-lengths that may be used for deriving the constants and testing the accuracy of series formulae, for comparing wave-lengths in terrestrial and cosmic sources, and for studying the changes in wave-length due to varying experimental conditions. For most purposes it is not a sufficient criterion that a wave-length be reproducible; it should also represent the fundamental vibration under standard conditions. Evidence that this end may be attained has been furnished by three lines of investigation: change of current strength, of arc length, and of constituents of negative pole. The basal assumption is that when no further variation in wave-length accompanies a change of any one of the three variables, the other two being held constant, the fundamental condition has been reached.

An entirely independent demonstration of pole-effect in the measures of Burns, Meggers, and Merrill and of the misleading interpretations that may follow from their use in statistical investigations is shown in Table X. The first column contains their

TABLE X

POLE-EFFECT SHOWN BY FICTITIOUS SUN-ARC DISPLACEMENTS

BURNS, MEGGERS, MERRILL	MT. WILSON		B, M & M *minus* MT. WILSON ARC	MT. WILSON SUN *minus* B, M & M	MT. WILSON SUN *minus* MT. WILSON ARC
	Arc	Sun			
5192.362....	.356	.359	+0.006	−0.003	+0.003
5205.609....	.606	.613	.003	+ .004	.007
5232.957....	.950	.954	.007	− .003	.004
5281.807....	.798	.802	.009	− .005	.004
5324.195....	.189	.193	.006	− .002	.004
5379.579....	.576	.578	.003	− .001	.002
5393.183....	.177	.182	+0.006	−0.001	+0.005
Means...	.....	.....	+0.006	−0.002	+0.004

For 17 flame lines, group *a*, mean difference Burns, Meggers, and Merrill *minus* Mount Wilson is −0.0004 A.

wave-lengths; the second and third, our determinations in the center of a 6-mm Pfund arc and in the sun made from high-dispersion spectrograms taken with a plane-grating spectrograph. The wave-lengths of these sensitive lines were referred to well-

established standards belonging to group *a*. It is obvious from the last two columns of Table X that opposite conclusions may be drawn from such comparisons, depending upon whether the arc wave-lengths used are contaminated with pole-effect, as in the Burns, Meggers, and Merrill measures, or whether they represent the more nearly fundamental values of the Mount Wilson data. In early work at Mount Wilson upon sun-arc comparisons the iron lines in the solar spectrum of groups *c* and *d* were displaced to the violet, and the lines of group *e*, abnormally to the red. It is now recognized that this arose from residual pole-effect in the center of the arc. Recent determinations with the 12-mm arc yield results of the same sign and magnitude for lines of the sensitive groups as for stable lines on the same plates as shown in Table XI. The

TABLE XI

SUN-ARC DISPLACEMENTS FOR UNSTABLE LINES BECOME NORMAL WHEN POLE-EFFECT IS ELIMINATED

λ	Fe Lines, Group *d*		λ	Fe Lines, Group *e*	
	Old, Pole-Effect	New, No Pole-Effect		Old, Pole-Effect	New, No Pole-Effect
5393.....	−0.006	+0.001	5364.....	+0.021	+0.011
5476.....	.004	.008	5367.....	.020	.008
5569.....	.002	.005	5364.....	.012	.008
5572.....	.002	.007	5383.....	.018	.006
5586.....	.006	.006	5404.....	.015	.006
5615.....	.000	.006	5410.....	.016	.012
5624.....	.008	.006	5415.....	.026	.005
5638.....	−0.008	+0.016	5424.....	+0.028	+0.008
Means..	−0.004	+0.007	Means..	+0.019	+0.008
9 stable lines, group *a*				+0.008	+0.008

disappearance of the abnormal behavior of such lines furnishes confirmatory evidence that pole-effect has been eliminated from the iron arc and indicates that the solar lines are produced under conditions in which pole-effect plays no part.

Not only is there evidence of pole-effect in the international iron wave-lengths, but it appears also in the determinations for other elements. In any given spectral region a "Standard R−I," Rowland *minus* international, for lines whose wave-lengths in sun

and arc are equal, may be obtained from stable iron lines by taking account of their sun-arc displacements. In Table XII are shown some results obtained by applying this criterion. The apparent displacement of the solar lines to the violet for lines showing positive pole-effect, indicated by negative values in the last column, and the abnormally large displacement to the red for lines with negative pole-effect are so similar to the results found for iron before the elimination of this disturbing influence that the abnormal sun-arc displacement shown in Table XII must be referred to errors in the arc wave-lengths rather than to conditions in the sun.

TABLE XII

POLE-EFFECT IN THE PUBLISHED WAVE-LENGTHS OF OTHER ELEMENTS

	Rowland λ	Pole-Effect	R−F	Standard R−I	Sun-Arc
Manganese...	4783.613	+0.012	0.181	0.187	−0.006
Fuchs*.....	4823.694	+0.011	0.175		−0.012
			R−Holtz		
Calcium.....	6102.937		0.201		−0.003
Holtz†.....	6122.434	+0.017‡	0.202	0.204	−0.002
	6162.390		0.194		−0.010
	3624.410		0.152		+0.016
	3647.707	−0.037‡	0.152	0.136	+0.016

* *Zeitschrift für Wissenschaftliche Photographie*, 14, 263, 1914.
† *Ibid.*, 12, 101, 1913.
‡ Whitney, *Astrophysical Journal* 44, 65, 1916.

A similar conclusion relative to the errors in the calcium wave-lengths of Holtz is reached when they are compared with the measurements *in vacuo* made by Crew and McCauley,[1] as in Table XIII. In this case the difference, Holtz *minus* Crew and McCauley, should be the pressure-displacement per atmosphere. Here again abnormal values are found bearing a relation to pole-effect similar to that observed for iron before its elimination from the iron data. As pole-effect practically disappears for iron *in vacuo*, it may be assumed with a high degree of probability that the

[1] *Astrophysical Journal*, 39, 29, 1914.

abnormal pressure-shifts indicated are due to errors in the wave-lengths of Holtz at atmospheric pressure rather than in those of Crew and McCauley *in vacuo*.

Data are not at hand for testing other published determinations. The tests here applied are simply illustrations of statistical comparisons already undertaken that have misled the investigator and of the uses to which such data may properly be applied. If the great labor expended upon the redetermination of wave-lengths is to be justified by results, due consideration should be given to the

TABLE XIII

FICTITIOUS PRESSURE-DISPLACEMENTS FOR CALCIUM DUE TO POLE-EFFECT

Series	Holtz	Crew and McCauley	Holtz *minus* C. and McC.	Pos. Pole *minus* Center Whitney
t.....	4092.690	.649	+0.041	
t.....	4094.983	.944	+0.039	+0.055
t.....	4098.575	.552	+0.023	
			+0.034	
T_2.....	6102.736	.716	+0.020	
T_2.....	6122.232	.216	+0.016	+0.017
T_2	6162.196	.177	+0.019	
			+0.018	
T_1.....	3624.106	.107	−0.001	
T_1.....	3630.739	.749	−0.010	
T_1.....	3630.958	.973	−0.015	−0.037
T_1.....	3644.757	.760	−0.003	
			−0.007	

elimination of pole-effect, a necessary preliminary to wave-length determinations. It is not too much to hope that the new data will be valuable for a generation at least, as the Rowland measures have been in the past, but, as the matter now stands, it is a question whether any of the published determinations of wave-lengths or of pressure-displacements for the more sensitive lines are sufficiently free from pole-effect to comply with the more and more exacting demands that will be made upon such fundamental data.

5. CONCLUSIONS

From the data presented above, the superiority of the Pfund arc 12 mm long, carrying a current of 5 amperes, as a source of standard spectrum lines appears to be established. Whether viewed from the standpoint of higher reproducibility, owing to increased sharpness of lines, or from that of attaining the fundamental wave-length, owing to the freedom from pole-effect, the advantage lies with this type of arc in preference to the arc specified by the International Committee. It is true that from λ 5650 to the red the decrease in brightness in the Pfund arc is sufficient to require rather long exposures when this source is used with high-dispersion apparatus; but from that point in the spectrum to the ultra-violet no difficulty is experienced. The international arc, on the other hand, is lacking in stable lines in the region λλ 5600–6000. To the red of λ 6027 there is a sufficient number of lines belonging to group *b*, which, being free from pole-effect, may be used as standards when observed in any form of arc.

The uniformity in the wave-lengths of the stable lines under varying conditions of arc is especially to be noted, along with the fact that even the most sensitive lines are unaffected in position by change of constitution of the negative electrode, while, under the set of working conditions developed, their wave-lengths exhibit remarkable constancy.

Attention should be directed also to the advantage of using the Pfund arc, under conditions similar to those described in this paper, as a source for working standards of intensity. The same conditions that favor constancy of wave-length also make for uniformity in the system of relative intensities, and the comparatively large range of working conditions permitted with this form of arc makes it much better suited to the purpose than an arc in which the intensity-gradient of a given line from center to pole is much steeper, as it is in the international arc. Obviously, if the intensity of a sensitive line is 5 or 10 times as great at the negative pole as it is at the center of an arc, it is highly important to reproduce with the greatest care the conditions under which the line is observed, if it is to be used as a reference standard of intensity. A slight change in the position of a slit which is at right angles to

the axis of the arc introduces an entirely new system of relative intensities in the case of a short arc, and a comparatively small error in focusing the image upon the slit will produce a similar result. Ample enlargement of the image, an achromatic projecting system accurately focused on the slit at a definite position and azimuth, preferably 90° to the axis of the arc at its midpoint, and a source in which the intensity-gradients from center to pole of even the sensitive lines are low, are some of the conditions which should be observed if the systems of relative intensity for spectrum lines in terms of the "arc" intensities used by different investigators are to have definite meaning.

It is a pleasure to acknowledge our indebtedness to the careful measurements and reductions carried out for us by Miss M. O. Burns, of the computing staff.

6. SUMMARY

1. The interference apparatus now in use at this observatory is described, and the methods of observation and reduction are explained.

2. By means of this instrument and a powerful plane-grating spectrograph quantitative data have been collected relating to the relative merits of three forms of iron arc when used as sources for standards of wave-length and of intensity.

3. The international arc has been shown to exhibit large pole-effects, not only at the negative pole, but in the central one-half, central one-third, and even in the central plane section.

4. The displacements at the negative pole as compared with the center are greater in this arc when the negative pole is below than when it is above.

5. In the Pfund arc with a carbon negative pole the wave-lengths of sensitive lines are the same at the negative pole and at the center.

6. The ordinary Pfund arc 6 mm long, carrying 6 amperes, has measurable pole-effect in its central region.

7. The Pfund arc operated with 5 amperes or less, at a length of 12 mm, is free from pole-effect in a central zone at least 1¼ mm wide.

8. The poorest lines in the iron spectrum, those of group *e*, when free from pole-effect, can be measured with high precision.

9. When pole-effect is eliminated, the wave-lengths of iron lines are found to be independent of the nature of the negative pole, whether this be of iron, carbon, copper, brass, or chromium steel.

10. The adopted wave-lengths of international standards belonging to groups *c* and *d* are shown to be influenced by pole-effect.

11. This influence enters to about the same extent into the values of wave-length recently published by other observers.

12. In the green part of the iron spectrum the pole-effect in Nos. 10 and 11 amounts to +0.006 A for lines of group *d*.

13. The sharpness of both stable and sensitive lines, when observed in a source free from pole-effect, is shown to be much greater than it is in the international arc.

14. The importance of eliminating pole-effect is shown by the misleading results obtained by using published data contaminated by this influence.

15. The abnormal sun-arc displacements formerly obtained for lines of groups *d* and *e* become of the same sign and order as for stable lines when pole-effect is eliminated.

16. Attention is directed to the importance of certain precautions to be observed when iron-arc lines are to be used for comparisons of intensity.

MOUNT WILSON SOLAR OBSERVATORY
June 1917

THE MINIMUM RADIATION VISUALLY PERCEPTIBLE[1]

By PRENTICE REEVES

"A determination of the least quantity of radiant energy capable of exciting the sensation of light could and probably by preference should be made in the laboratory by a direct method." This statement by H. E. Ives[2] led the writer to start on the study of this subject. Ives computed the value for the *minimum visibile*[3] from data at hand, and since then H. N. Russell[4] has supplemented the work by introducing data inaccessible to Ives. The data used by Ives on the mechanical equivalent of light and the stellar magnitude of the standard candle were well established, and the data offered by Russell were on the pupilary area of an eye adapted to the dark, and on the stellar magnitude of the faintest visible objects. These values used by Russell are respectively twice and one-tenth those used by Ives, so that the corrected value of the *minimum visibile* is just one-fifth that of Ives. By using a direct method in the laboratory, where all physical stimuli were under the control of the experimenter, the uncertainties of stellar observations were largely overcome. Another advantage of this paper is the use of measurements taken of the observers' pupils. The luminosity-curves of the observers show each observer to have a normal area-curve.

The apparatus used was a modification of the visual sensitometer used by P. G. Nutting,[5] and a light-proof lamp-house with decimal neutral filters. The diagram, Fig. 1, shows the scheme used to control the intensity of the light in the visual sensitometer. The

[1] *Communication No. 51*, from the Research Laboratory of the Eastman Kodak Company.

[2] "The Minimum Radiation Visually Perceptible," *Astrophysical Journal*, **44**, 124–127, 1916.

[3] This term was used by Russell to indicate the threshold value for perceptible radiation.

[4] "The Minimum Radiation Visually Perceptible," *Astrophysical Journal*, **45**, 60–64, 1917.

[5] "The Retinal Sensibilities Related to Illuminating Engineering," *Trans. of the Illum. Eng. Soc.*, **11**, 1–21, 1916.

light-source (N) is focused on the slit (S) by means of the lens (L), and the stimulus is illuminated through the opal glass window (O). The size of the stimulus is controlled by screening the window and in this way any desired aperture is obtained. The intensity of the light is controlled by means of the absorbing wedge (W), which is operated by the experimenter and slides on machined metal ways over the slit. By reading the brightness of the window without the wedge and by knowing the density of the wedge at any setting the transmitted light is computed from the equation

$$\log I_0 = \log I - D,$$

where I is the brightness of the window with no wedge, D is the density of the wedge before the slit, and I_0 is the reduced brightness.

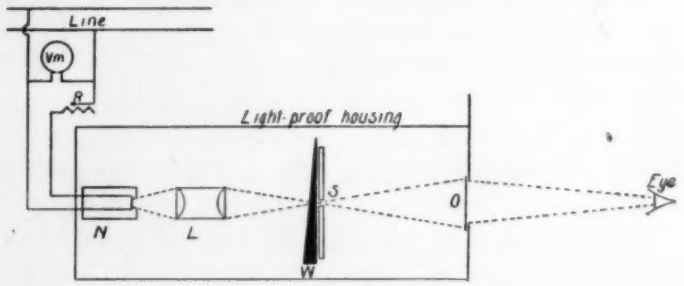


FIG. 1.—Stimulus control

The wedge was made by coating neutrally dyed gelatine on plate glass in a wedge-shape, so that the density varies from about 0.03 to 7.5, i.e., the percentage of the incident light transmitted varies from about 92 to 3.2×10^{-6}. When the dyed gelatine dried, a cover-glass was bound over it, as the unprotected gelatine soils with handling and is susceptible to moisture. The bound wedge is carefully calibrated and checked from time to time. A millimeter scale is mounted on the wedge, and the portion of the wedge before the slit is read from a pointer. A device was placed on the apparatus for recording the position of the wedge when work was done in darkness and the records could be read later.

The observer used a head-rest for all observations, so that the distance, fixation, etc., would be as nearly constant as possible. To approximate conditions of stellar observations, a stimulus 1 mm in

diameter viewed at a distance of 3 m was used in the first part of the experiment. With this arrangement of the apparatus readings were also taken at distances of 1.5 m and 35 cm. Several light-sources were used in this part of the work, but the results are very little different from one another. The results taken under the same conditions from day to day vary through wide ranges, and this characteristic has been found by the writer and other experimenters to hold for all threshold work with visual stimuli.

When observations were made with this apparatus, all adjustments were made and the observer then remained in total darkness for at least fifteen minutes before making any observations. The "star" was then exposed at a brightness just above the visual threshold and slowly dimmed by moving the wedge until the threshold was reached and the position of the wedge was recorded. The wedge was then set so that the "star" could not be seen and moved slowly until the "star" just became visible and this position recorded. This procedure was repeated several times, and the average of the several readings was taken as the threshold value for that series. (When the lamp in a lamp-house was employed, a 1-mm "star" was used and the intensity of the light was controlled by a set of decimal filters. By reading the brightness with no filters and knowing the filter-densities the investigator could compute the brightness of the star. The procedure in this case was the same as before.)

When the experiment was first started, the diameter of the pupil was measured by means of a glass wedge. The width of the wedge was 1 mm at the small end and 11 mm at the wide end and was 10 cm long, so that the wedge increased 1 mm in width for each centimeter of length. The wedge was also thicker at the wide end, making it prismatic, and an object viewed through it was displaced. If the wedge was held before the eye so that the pupil was only partly covered, two images were seen, one over the other, as one image came through the glass and was displaced and the other was the normal image seen around the wedge. By shifting the wedge a position could be reached where the sides of the wedge were both tangent to the pupil and only the displaced image was seen. A slight shift of the wedge introduced the second image, and

the pupilary diameter was obtained by measuring the width of the wedge where the second image just disappeared. To get the diameter of the pupil when adapted to darkness, the small "star" was exposed at an intensity slightly above the threshold and was used as an object for the wedge. The amount of light used was considered too small to cause a contraction of the pupil. The results from this wedge show the average maximum pupil to be 6.8 mm, a value too small, according to some experiments, although in agreement with other results when using faint sources of light. T. H. Blakesley,[1] by using a faint light-source, found that his pupil varied in diameter from 6.74 to 7.20 mm, the higher values being less frequent. A probable explanation for the failure of the method of the glass wedge may be that for the light-source to be seen around the wedge a relatively large area of the pupil must be uncovered, in order to allow sufficient energy to reach the retina to give rise to a sensation, and, as a result, the width of the wedge before the pupil is much less than the pupilary diameter. Or it may be a question of the distribution and sensitiveness of retinal elements at the extreme of the periphery.

In order to get a better method for recording the pupilary diameter, it was decided to take instantaneous flash-light pictures of the eye adapted to the dark. A millimeter scale was pasted near the eye in the same plane as the pupil, the subject was seated near the camera, so as to secure a large image, and after sufficient adaptation the picture was taken. The average maximum diameter of the writer's pupil as taken from several photographs is 8.3 mm. The results show that the diameter of the pupil is not constant, even when the level of brightness of the environment remains quite fixed and the eye is fully adapted to the dark. The photographs were taken on different days and also on the same day, with sufficient interval between exposures for the eye to return to normal conditions. Photographs of eyes of other observers show just as wide variations, and the average maximum diameters also show individual differences. These fluctuations in diameter were found to hold throughout the range of intensities from total darkness to the reflection from white drawing-paper in full sunlight. The

[1] "A Means of Measuring the Apparent Diameter of the Pupil of the Eye in Very Feeble Light," *Phil. Mag.*, **20**, 966–969, 1910.

writer and Mr. Julian Blanchard took photographs of the eye at the two extreme intensities and at six intermediate intensities for both eyes exposed to the sensitizing brightness and for only one eye exposed, the other eye being closed. (These data will be published as soon as the experiments can be completed.) The results do not agree with those published by W. H. Steavenson,[1] who used five normal subjects between the ages of twenty-one and twenty-five years and found an average diameter of 0.336 inches or 8.5 mm, with an average deviation from the mean of only 3 per cent. This average seems to be higher than the writer's results, and the variation much too small. The writer used three subjects trained in dark-room observations and expects to photograph several other subjects in the near future.

Nutting says: "The image of the pupil formed in front of the eye by the aqueous humor and the cornea is the entrance pupil of the optical system, and, since the position of the iris varies with the accommodation, so will this entrance pupil."[2] The relative diameter of pupil and image focused at 25 cm is 1.02, and, as this is about the distance used for the photographs, the correction may be safely neglected.

Table I shows the data obtained for the 1 mm "star" at a distance of 3 m with the different light-sources, and the variation from one source to another is seen to be no more than that between results from the same source. The average of all observations was used in computing the final results. Table II shows data for the "star" at 1.5 m and 35 cm. Table III shows the comparative results of three observers under the same conditions. Table IV contains a summary of results and the computed value of the *minimum visibile*.

If we let B = normal candle-power per square centimeter of source, assuming the inverse square law and a point-source, the flux through 1 sq. cm on the axis at the eye will be

$$\frac{SB}{R^2} \text{ lumens,}$$

[1] "Aperture of Eye-pupil," *Journal of B. A. A.*, **26**, 303, 1916.

[2] *Outlines of Applied Optics*. Philadelphia: P. Blakiston's Son & Co., 1912. Pp. xi+234.

and the flux through the pupil of area A will be

$$Fp = \frac{SBA}{R^2} \text{ lumens.}$$

Now B = Lumens π and $A = \pi r^2$, so

$$Fp = SL\frac{r^2}{R^2},$$

where Fp is the flux through the pupil; S, the area of the "star" in square centimeters; L, the brightness of the star in lamberts; r, the radius of the pupil; and R, the distance of the eye from the "star." If this equation is then multiplied by the mechanical equivalent of light, M (using Ives's value of 1.59 ergs per sec. per sq. cm), we get the equation for the *minimum visibile* as

$$\text{Least perceptible radiation} = SLM\frac{r^2}{R^2} \text{ ergs per sec.}$$

TABLE I

1 mm "star" at distance of 3 m

100-watt carbon filament lamp, 111 volts, 0.904 amps. Color-match with standard

Sensitometer with Wedge	Lamp-House with Filters
0.0065 ml*	0.0102
.0053	.0180
.0126	.0144
.0093	.0155
0.0074	.0161
	.0142
Mean 0.0082	.0153
	0.0072
	Mean 0.0064

Sensitometer with wedge

Nernst glower No. 1, 27 mm long, av. diam. 0.63 mm, 156 volts, 0.4 amp.:

Mean = 0.0088

Small carbon lamp, 116 volts, 0.530 amps for color-match:

Mean = 0.0095

40-watt tungsten, 116 volts:

Mean = 0.0045

Nernst No. 2, 36 mm long, av. diam. 0.66 mm, 220 volts, 0.2 amps.:

Mean = 0.0072

Mean of all = 0.0072

* All results are in millilamberts, one thousandth of a lambert. It has been officially adopted by the Illuminating Engineering Society and was defined by the Committee of Nomenclature and Standards (*Trans. Illum. Eng. Soc.* 10, 642, 1915,) as "the brightness of a perfectly diffusing surface radiating or reflecting 1 lumen per sq. cm," that is, in accordance with Lambert's cosine law: 1 ft.-candle = 1.076 ml, 10 meter-candles = 1 ml.

In Table IV the writer's threshold is seen to be 17.1×10^{-10} ergs per sec., the average for three observers is 19.5×10^{-10}, as compared with Ives's computed result of 38×10^{-10} and Russell's experimental result of 7.7×10^{-10} ergs per sec. (Nagel[1] reviews the earlier work done in this field and gives as his own result for the smallest quantity of energy sufficient for stimulation 31.6×10^{-10} ergs per sec. but does not give details of procedure for comparative purposes.)

TABLE II

1 mm "star" at distance of 1.5 m.
100-watt carbon lamp

Sensitometer with Wedge	Lamp-House with Filters
0.0029	0.0028
0.0034	0.0030
0.0026	0.0020
	0.0022
Mean=0.0030	0.0016
	0.0026
	Mean=0.0024
Mean of all=	0.0026

1 mm "star" at distance of 35 cm
100-watt carbon lamp

Sensitometer with Wedge	Lamp-House with Filters
0.00015	Mean=0.00027
0.00026	
0.00029	
Mean=0.00023	
Mean of all=	0.00024

TABLE III

1 MM "STAR" AT 3 M., NERNST NO. 2

Observer	Threshold	Diameter of Pupil
1	0.0072 ml	8.3
2	0.0100	8.4
3	0.0092	7.4
Means	0.0088	8.0

At first the wide variations in the separate threshold values in Table I seem too great, but let us consider some of the factors which influence the threshold determination. The threshold varies with

[1] Helmholtz *Handbuch der physiologischen Optik*, 3. Aufl., 1911, Band 2, p. 289.

the pupilary diameter, and this we found was not constant. When the stimulus is just at the threshold, we must consider the fluctuations due to attention and fatigue. Ideo-retinal light, probably caused by the retinal circulation, is a decided hindrance to accurate observations and at times compels the observer to stop all procedure. After-images often persist after the stimulus has been removed and are likely to recur so as to be confused with the stimulus itself. Involuntary movements of the eye make the conditions of visual fixation variable, and any change in the area of the retina used would alter the result obtained. Probably the greatest factor is

TABLE IV

	Threshold	Energy at Eye	Area of Pupil	Energy Entering Eye
			sq. cm	ergs/sec.
Writer	0.0072 ml	3.16×10^{-9}	0.54	17.1×10^{-10}
Mean of 3 observers	0.0088 ml	3.89×10^{-9}	0.50	19.5×10^{-10}
Ives's data	$6^{M}0$	1.35×10^{-8}	0.28	38.0×10^{-10}
Russell	$8^{M}5$	1.35×10^{-9}	0.57	7.7×10^{-10}

the physiological condition of the observer from one day to the next. To determine an extreme of this factor, the writer took observations when affected by a severe cold, and the threshold was raised so that the values could not be used at all. As these factors are not subject to control, the only method is to take as many observations as possible over a wide range of time and to accept the general average as the threshold value.

These difficulties make the problem of determining the length of time required for such luminous intensities to produce a sensation almost a hopeless one. The writer is now collecting data on this subject, as well as on the problem of the visibility of large and small images of the same total intensity, and they will be published later.

The writer wishes to acknowledge the assistance of Messrs. L. A. Jones, Julian Blanchard, and Milton Fillius in the preparation of this paper.

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PRELIMINARY EXAMINATION OF THE PLANETARY NEBULAE FOR PREFERENTIAL MOTION

By C. D. PERRINE

As it is generally believed that the planetary nebulae belong to our stellar system and as their average velocities are high, the question is raised whether, like the stars, there may be a tendency among them to motion in the direction of the ellipsoidal axis.

The most valuable data for such a purpose, viz., radial velocities and proper motions, are meager indeed. However, it seemed worth while to make the attempt, as the total number of these objects appears to be very limited, and, were complete data available for all such objects known, the weight of any conclusions would probably be very inferior to corresponding results for the stars. We also have any evidence which may be furnished by the distribution of these objects in the sky.

For this investigation there were available individual radial velocities for but fifteen of these objects, the classical thirteen by Keeler[1] and two by Campbell.[2] As the two by Campbell are exceptionally high, the determination of prolateness was limited to Keeler's list.

But two proper motions were available, viz., by H. D. Curtis.[3]

The available data are collected in Table I. v' are the radial velocities corrected for a solar motion of 20 km toward $18^h, +30°$. The velocities of Nos. 6644 and 4732_2 are by Campbell.

It is interesting to note that the two with very high velocities by Campbell are very close to the southern vertex of the ellipsoid and that both are stellar. It is also worthy of note that all three stellar objects of Keeler's list have large positive velocities, two of them being the highest on his list, and that these two are also close to the same vertex. With the exception of these stellar objects, the others are large and bright.

[1] *Publications Lick Observatory*, 3, 201.

[2] *Proceedings of the National Academy of Sciences*, 1, 9, 1915.

[3] *Ibid.*, 1, 10, 1915.

PROLATENESS

As stated above, the excessive velocities of Campbell's two nebulae were not used in the determination of prolateness. If we adopt limits of 50° from the vertices for the major axis and of 60° to 90° from the vertices for the minor axis, there are 8 nebulae yielding a velocity of 32.0 km for the major axis and 5 yielding a value of 18.6 km for the minor axis. These values give a prolateness of 1.72, which agrees well with the value given by the stars of large proper motion.

TABLE I

DATA FOR PLANETARY NEBULAE

N. G. C.	Description	α 1900.0	δ 1900.0	Obs. v	v'
1535.....	vB	$4^h\ 9^m36^s$	−13° 0′	−10.2 km	− 27.6 km
3242.....	vB	10 19 58	−18 8	+ 6.0	− 4.0
6210.....	vB	16 40 18	+23 59	−34.4	− 15.6
6309.....	B	17 8 26	−12 48	−51.6	− 37.8
6543.....	vB	17 58 35	+66 38	−64.8	− 48.8
6572.....	vB	18 7 14	+ 6 50	− 9.7	+ 8.3
6644.....	Stellar	18 26 24	−25 13	.....	+202
4732_2.....	Stellar	18 27 54	−22 43	.....	−141
6790.....	B Stellar 9.5	19 17 52	+ 1 19	+48.5	+ 64.7
6818.....	B,vS	19 38 20	−14 24	−16.8	− 4.2
6826.....	B,pL	19 42 7	+50 17	− 5.3	+ 12.5
6891.....	Stellar 9.5	20 10 25	+12 26	+40.8	+ 57.0
6905*.....	B //	20 17 56	+19 47	.....	.....
7009†.....	vB ///	20 58 44	−11 46	−49.8	− 39.8
7027.....	Stellar 8.5	21 3 18	+41 50	+10.2	+ 25.6
7662.....	vB ///	23 21 5	+41 59	−11.5	− 2.5

*μ=0″.056. †μ=0″.054.

The amount of data is extremely limited, and the result is therefore of small weight. As far as it goes, however, it is definitely in favor of an excess of motion in the general direction of the ellipsoidal axis. Little information can be drawn from the only two measures of proper motion available, further than that they appear to be just within the limits at which preferential motion was found among the stars.

DISTRIBUTION

The distribution of these objects seems to furnish stronger evidence of a preference for the direction of the ellipsoidal axis. In

the diagram published by H. D. Curtis[1] are seventy-three of these objects north of 33° south declination. Two-thirds of these are within 50° of the ellipsoidal vertices, or about one-third of the total area of the sky.

Curtis points out in the same note that the smaller of the planetaries show the greater affinity for the galactic plane. His diagram shows that the smaller ones have an apparent preference also for the ellipsoidal vertices. This and the generally high velocities

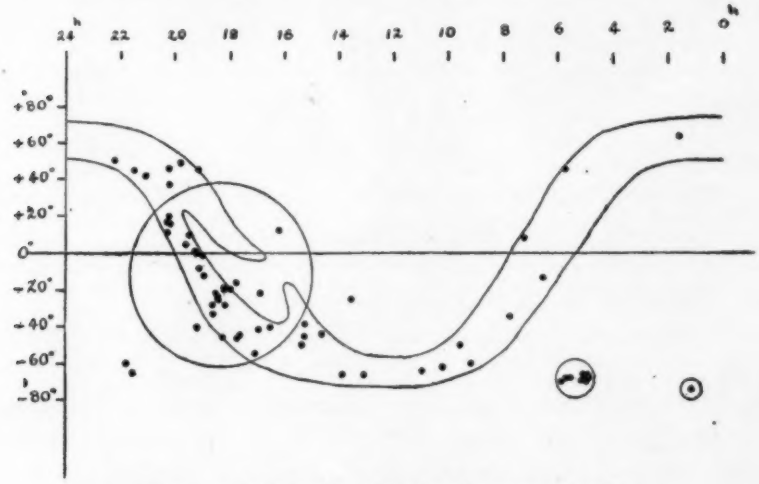

FIG. 1. Distribution of stellar planetary nebulae

of the five stellar planetaries of the foregoing short list caused me to chart the positions of the sixty-five stellar planetaries of the three *N.G.C.* catalogues for the entire sky. The resulting distribution as shown in the accompanying diagram is very suggestive. Not only do these objects show a striking affinity for the galaxy, as pointed out by Curtis, but they also show a most decided preference for the 18h ellipsoidal region and an equal avoidance of the opposite region of sky. Their affinity for the galaxy is not confined alone to its plane, but extends to the actual branches. This is particularly striking in the Sagittarius-Ophiuchus-Aquila region (the region of the great bifurcation), where for a considerable space, and that richest in planetaries also, they are wholly confined to the southern

[1] *Publications of the Astronomical Society of the Pacific*, **29**, 53, 1917.

branch, the northern, terminant streamer not having any so far as known.

Even more striking, if possible, are the groupings of these stellar planetaries in the Magellanic Clouds. Taken in connection with the appearance and known structure of the Clouds, this evidence of the stellar planetaries would seem to justify, not only the conclusion that the Magellanic Clouds are similar in constitution to the Milky Way, but the strong presumption that they are in reality more or less isolated parts of the galaxy.

Considering the evidence as a whole, there seems no room to doubt that the planetary nebulae are members of our own stellar system.

CONCLUSIONS

1. The planetary nebulae show a preference for the regions of the vertices of ellipsoidal motion, particularly that near 18^h. The few radial velocities of these bodies which are available also show greater motion in the direction of this axis.

2. The planetary nebulae undoubtedly belong to our stellar system.

3. The restriction of the stellar planetary nebulae to the Milky Way and Magellanic Clouds indicates, if in connection with the appearance and other known facts of the constitution of these several bodies it does not establish, a very close relationship between the Clouds and the Milky Way.

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April 8, 1917

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